Physics Aptitude Test (PAT) Unofficial Solutions: 2019

University of Oxford Admissions Test

Physics, Engineering, Materials Science

Solutions by PhysOx Initiative
2020

Contents

P.I

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1 Foreword

The solutions provided here are by no means an official set of answers; our aim was to provide unofficial solutions in order to provide those studying for the PAT with the means to check their answers as they attempt past paper questions. There is no mark breakdown but we aimed to provide detailed explanations on how to solve the questions and develop a good intuition for them. These solutions have been compiled by our team of Oxford Physics graduates who have all taken the PAT and our team members also have experience in marking the PAT and/or running the PAT summer programmes coordinated by the University of Oxford.

Some general tips for questions:

1) Try and **keep your calculations in terms of symbols and letters** until you see your calculations simplifying considerably by substituting numbers. This will genuinely reduce errors in your work and make it way easier for the marker to understand your thought process.

2) Give your **variables reasonable names**, for example don't call your initial velocity something like v_u and final velocity something like v_v, as you will definitely confuse the life out of everyone looking at your work not to mention yourself.

3) Your teachers may say this a lot and many of you probably ignore it but **drawing diagrams really helps!** Sometimes the best way to deal with paragraphs of information is a simple drawing which has all the important bits - it will also save you a lot of time!

4) Always **show your working** so it does not look like you picked an answer out of thin air especially for the longer answer questions so it's easier to pick up on exactly where you went wrong if you do go wrong. Fair enough you may have had a moment of next level inspiration but a couple of lines (or words here and there) just outlining your way of thinking really helps. That said, it doesn't have to be an essay!

5) Don't feel the need to rush, **relax** yourself and approach the questions. If you get really stuck on one, don't get too put off, just skip it and come back to it later if you get time.

2 Section A: Multiple Choice Questions

2.1 Answers

1	D
2	A
3	C
4	A
5	A
6	A
7	E
8	A
9	D
10	E
11	C
12	B

Table 1: Multiple Choice Answers.

2.2 Answers Explained

2.2.1 Qu1: Series

This is a geometric series with common ratio $r = -1/3$.
The next term is then just:

$$-12 \times \frac{-1}{3} = 4$$

. The answer is D.

2.2.2 Qu2: Simultaneous Equations

Simultaneous equations with logs. First we note that $log32 = 5log2$. Then we just do the following:

$$logx + 2logy = 5log2$$

$$-(logx - logy = -log2)$$

$$3logy = 6log2$$

$$logy = 2log2 = log4$$

$$y = 4$$

Then apply back into one of the equations.

$$logx = -log2 + 2log2$$

$$= log2$$

So $x = 2$ and the answer is A.

2.2.3 Qu3: Gravitational Potential

This question is just a bit of rearranging but the one thing we need to know is the form of the gravitational potential:

$$\Phi_{grav} \propto r^{-1}$$

So that $n = -1$.

Then it's just a bit of algebra.
We have been given:

$$2\langle T_{tot}\rangle = n\langle V_{tot}\rangle = -\langle V_{tot}\rangle$$

Now we use the definition of total energy.

$$\langle E_{tot}\rangle = \langle T_{tot}\rangle + \langle V_{tot}\rangle$$

$$= \frac{-1}{2}\langle V_{tot}\rangle + \langle V_{tot}\rangle$$
$$= \frac{1}{2}\langle V_{tot}\rangle$$

The answer is C.

2.2.4 Qu4: Ratios

We are given the acceleration due to gravity:

$$g = \frac{GM}{R^2}$$

There are ofcourse many ways to do this but perhaps the best way is to look at ratios and proportionalities. Let the constants and variables in the second universe be distinguished as primed variables G′, R′, M′.

We are told:

$$G' = 2G$$

Also

$$R' = R/2$$

Which implies that the volume (which is $\propto R^3$) behaves as:

$$V' = V/8$$

We then can say that as $M = \rho V$:

$$M' = 2\rho\frac{V}{8} = \frac{M}{4}$$

Now applying these back into the equation for gravitational acceleration:

$$g' = \frac{2G.M/4}{(\frac{1}{2})^2 R} = 2g$$

Therefore the answer is A.

2.2.5 Qu5: Trigonometry

We want this equation in terms of one type of trigonometric function ideally so let's use an identity. If we start with everyone's favourite:

$$sin^2\theta + cos^2\theta = 1 \tag{1}$$

We can divide both sides by $cos^2\theta$:

$$tan^2\theta + 1 = sec^2\theta$$

Now our equation becomes:

$$tan^2\theta + \alpha tan\theta + 1 = 0 \quad (2)$$

This is just a quadratic! Apply the quadratic equation and we are there:

$$tan\theta = \frac{-\alpha \pm \sqrt{\alpha^2 - 4}}{2} \quad (3)$$

For real solutions the discriminant $\alpha^2 - 4 \geq 0$ so this implies:

$$\alpha \leq -2$$

$$\alpha \geq 2$$

The answer is A.

2.2.6 Qu6: Probability

The probability I get different colours is given by:

1. The probability that I pick a red ball, then a blue
OR
2. The probability I pick a blue ball, then a red.

Only caveat is the first time I pick from (r+b) balls whilst the second time I've removed one ball so I pick from (r+b-1). So the answer is either A or B.

Probability 1:

$$\frac{r}{r+b} \bullet \frac{b}{r+b-1}$$

Probability 2:

$$\frac{b}{r+b} \bullet \frac{r}{r+b-1}$$

And as we have the OR condition we sum the two different possibilities (which are the same) and end up with A.

2.2.7 Qu7: Combinations

10 fingers and there are 2 possibilities per finger so we have 2^{10} possibilities.

These kinds of questions might be a bit confusing at first so dial down the numbers and try out a simper scenario as a toy model of your idea. In this case considering the number of combinations you can get for 1 finger (2), 2 fingers (4 as 2 curled, 2 stretched, finger 1 curled finger 2 stretched or finger 2 curled finger 1 stretched) and even 3 (8 if you counted up the possibilities again) gives you a good idea of what the answer could be.

2.2.8 Qu8: Odd and Even Functions

You don't need to calculate these integrals because the limits are symmetric! This means that if we have an odd function (rotationally symmetric) we should get cancellation whereas even functions should give a non-zero answer. This is summarised below:

1. Odd functions $= 0$.
$$-f(x) = f(-x) \tag{4}$$
2. Even functions $\neq 0$.
$$f(x) = f(-x) \tag{5}$$

So based on these definitions, I_2 and I_3 should be non-zero corresponding to option A.

2.2.9 Qu9: Current-Carrying Wire

If the distance between the two wires are D then the radial distance of the point we are interested in is $D/2$ away from both wires. The magnitude of the $\vec{B}$ field vectors are therefore:

$$B = \frac{\alpha I}{\frac{D}{2}}$$

$$B_2 = \frac{\alpha I_2}{\frac{D}{2}}$$

If we want double the flux density, we need the magnitudes B and B_2 to be the same which is clearly the case as long as:

$$|I| = |I_2|$$

But we need to be careful with signs. Here's where a diagram comes in handy! If we assume currents going in the same direction, then from the Right-hand Grip Rule the field lines in the centre are in opposing directions so we need the two currents going in opposite directions. So our answer is in fact:

$$I_2 = -I$$

Or D.

2.2.10 Qu10: Lunar Phases

Again diagrams really help with this.

When we see a full moon, this means the sun is shining on the face of the moon that the Earth can see (i.e. Earth is between the Moon and the Sun). This means if I was on the lit-up face of the Moon, I would see the Sun behind the Earth and the face of the Earth facing me should be completely dark and so we expect a New Moon or E.

In this application of phases of planetary bodies we usually experience a classic rule of opposites.

2.2.11 Qu11: Resistance in Circuits

We need to know two things:

1. Resistances in Series Add up: $R \Uparrow$

$$R^s_{tot} = \sum_i R_i \tag{6}$$

2. Resistances in Parallel, Inverses add up: $R \Downarrow$

$$\frac{1}{R^p_{tot}} = \sum_i \frac{1}{R_i} \tag{7}$$

Now the higher the resistance, the lower the brightness. And more series elements increases the resistance, reducing the brightness. Therefore we expect C to be correct but we can check just in case. Branch A:

$$R_A = R + R/2 = 3R/2$$

Branch B:

$$R_B = 2R$$

Both:

$$R_{AB} = \frac{1}{\frac{2}{3R} + \frac{2}{3R}} = \frac{6R}{7}$$

So as expected resistances decrease as : B is closed, A is closed, AB are both closed. And as a result the brightness increases in this order so C.

2.2.12 Qu12: Standing Waves and Harmonics

This is your standard harmonics question. We are looking for the fundamental frequency of this system.

The open end forms an antinode and the closed end is a node so the longest wavelength possible is for:

$$\lambda = 4L$$

Now combining with the almighty wave equation:

$$v = f\lambda$$

$$v = 4Lf_{min}$$

and then rearranging for L we obtain B.

3 Section B: Long Answer Questions

3.0.1 Qu13: Graph Sketching

a) First let us explicitly write down the curves we need to sketch. For $n = 0, 1, 2, 3$:

$$y = 1$$
$$y = 1 + x$$
$$y = (1 + x)^2$$
$$y = (1 + x)^3$$

Points in common to all graphs is the point $(0, 1)$.

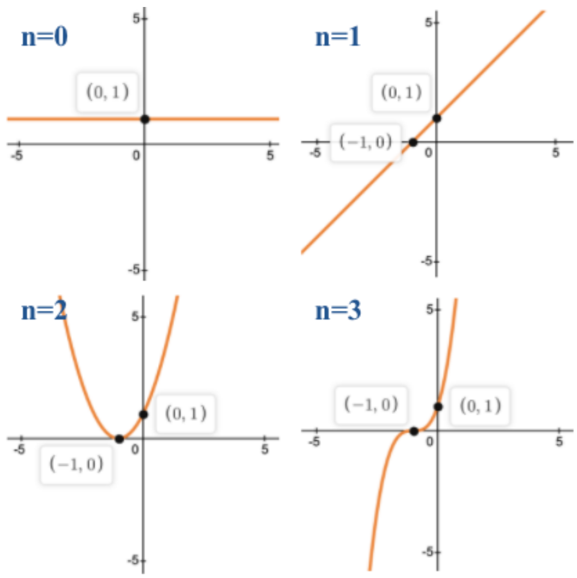

Figure 1: Qu13

b) Features in common to the graphs for $n > 1$:

1. The point $(-1, 0)$ is the x intercept.

2. Point $(-1, 0)$ is also a turning point and both graphs only have this one turning point.

3. As $|x| \longrightarrow \infty$, $|y| \longrightarrow \infty$.

4. Both curves are neither odd nor even functions OR both graphs exhibit some

type of symmetry (rotational for $n = 3$ and reflection for $n = 2$) about the line $x = -1$.

3.0.2 Qu14: Radioactive Decay

a) Half life $t_{\frac{1}{2}}$ occurs when:

$$\frac{N_A}{N_{A0}} = \frac{1}{2}$$

as N_A is the number of atoms left in the sample after a given time t and N_{A0} is the number of atoms of A initially at $t = 0$. Solving this:

$$\frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}} = \frac{ln2}{\lambda}$$

b) We are given the initial population of B and are told that it is stable but the population of A decaying into B also contributes to the total population of B. So the population:

$$B(t) = B(t = 0) + A_{decayed}$$

The decayed population of A is the initial A population ($A0$) - number of A atoms left:

$$A_{decayed} = N_{A0}(1 - e^{-\lambda t})$$

$$N_B = N_{B0} + N_{A0}(1 - e^{-\lambda t})$$

c) The buzz word here is ratio so let's write down the expression for the ratio we want:

$$\frac{N_B}{N_A} = \frac{N_{B0} + N_{A0}(1 - e^{-\lambda t})}{N_{A0}e^{-\lambda t}}$$

Now we are told that at $t = 0$:

$$\frac{N_{A0}}{N_{B0}} = \frac{1}{x} \implies N_{A0} = xN_{B0}$$

And we want the time when the ratio is reversed to:

$$\frac{N_B}{N_A} = x$$

Let's combine all of this information and solve. This questions is fundamentally not too bad but not messing up the algebra in all our haste can be potentially problematic, so take a deep breath and dive in!

$$\frac{N_B}{N_A} = \frac{N_{B0} + xN_{B0}(1 - e^{-\lambda t})}{xN_{B0}e^{-\lambda t}} = \frac{1 + x(1 - e^{-\lambda t})}{xe^{-\lambda t}} = x$$

Simplifying this:

$$x = \frac{1}{x}e^{\lambda t} + e^{\lambda t} - 1$$
$$x^2 = e^{\lambda t} + xe^{\lambda t} - x$$
$$x(x+1) = e^{\lambda t}(x+1)$$
$$x = e^{\lambda t}$$
$$t = \frac{lnx}{\lambda}$$

This looks pretty nice and resembles the equation for the half life which sort of makes sense as technically speaking we are requiring and solving for a particular fraction of decayed atoms after a given time interval.

3.0.3 Qu15: Refraction

a) We are told that $n_g > n_l > n_a$ so from Snell's Law:

$$\theta_1 < \theta_i < \theta_2$$

Where θ_i is the incident angle (to the normal) in the liquid, θ_1 is the angle to the normal in the gas phase and θ_2 is the angle in the air phase.

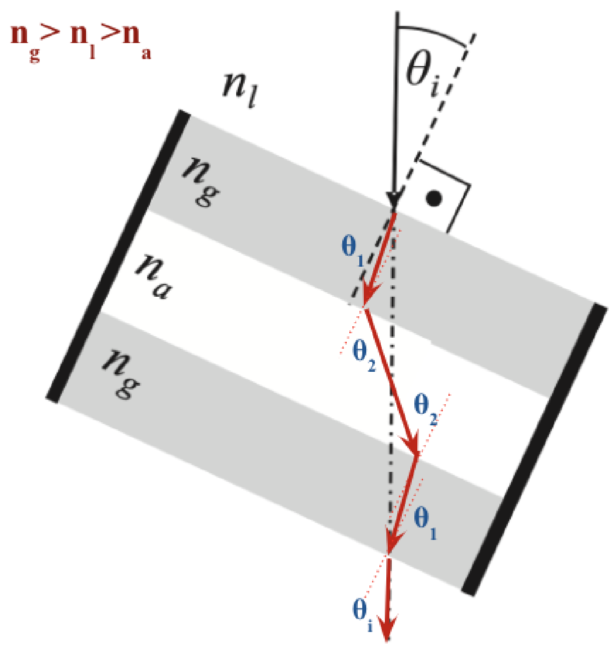

Figure 2: Qu15

b) As you increase the angle θ_i the angle at which the light is observed will change. Can use Snell's Law to find the relationship between the angles:

$$n_l sin\theta_i = n_g sin\theta_1$$

$$n_g sin\theta_1 = n_a sin\theta_2$$

As $\theta_1 < \theta_i < \theta_2$ eventually θ_i will increase to a value such that there is total internal reflection at the gas-air interface at which point the light will disappear/ no light will reach the other end.

$$n_g sing\theta_1 = n_a$$

$$n_l sin\theta i = n_g sin\theta_1 = n_a$$

so

$$sin\theta_i = \frac{n_a}{n_l}$$

c) Improvements to the measurement: From one of the conditions derived in (b):

$$n_g sin\theta_1 = n_a$$

If we use a gas with a higher refractive index then from

$$sin\theta_i = \frac{n_a}{n_l}$$

we have more control over θ_i allowing better estimate of n_l.

We can also use monochromatic light to make our observations clearer.

A CCD can be used to enable greater accuracy in measurement of the light.

3.0.4 Qu16: Hydrogen Atom Energy Levels

a) Planck's formula:

$$E = \frac{hc}{\lambda} \tag{8}$$

b)

$$\frac{hc}{\lambda} = E_p - E_q = -hcR(\frac{1}{p^2} - \frac{1}{q^2})$$

$$\lambda = \frac{1}{R}(\frac{1}{q^2} - \frac{1}{p^2})^{-1} = \frac{1}{R}\frac{p^2q^2}{p^2 - q^2}$$

c) At first glance it could be tempting to freak out about the trial and error process that seems inevitable but there is a shortcut to all that. If we look at the formula for λ again:

$$\lambda = \frac{1}{R}\frac{p^2q^2}{p^2 - q^2}$$

Cell No.		λR	
p	A	B	C
q+1	1.334	44.45	81.86
q+2	1.126	28.81	51.07
q+3	1.067	23.77	41.05
q+4	1.042	21.33	36.18
q+5	1.029	19.94	33.35

We can see that this formula converges for large values of p.

$$\lambda_{large p} R \longrightarrow \lim_{p\to\infty} \frac{p^2q^2}{p^2-q^2} = \frac{p^2q^2}{p^2} \implies q^2$$

Ofcourse the p values are no where near infinity in the data we have been given but it is getting bigger so we should still notice some sort of convergence which might help us. Below I have a table of all of the $\frac{p^2q^2}{p^2-q^2} = \lambda R$ values. Realistically you only actually need the first row (as a check) and the last row (to do the calculation). The "trick" is to set the values in the last column equal to the q^2 and see if we can get a number that works with the first row entry (and also the other entries as an extra check).

Now let's apply what we discussed!

a) For set A:

$$q^2 \approx 1.029$$

so

$$q = 1$$

Check with the first row entry:

$$\frac{2^2 1^2}{2^2 - 1^2} = \frac{4}{3} \approx 1.334$$

Awesome it worked!

b) For set B:

q^2 somewhere around 19. We know that the number must be a square number (as q must be an integer) which is below 19 (as values converge to a smaller integer each time). Let's try 16.

$$q \approx 4$$

Check with first row entry:

$$\frac{4^2 5^2}{5^2 - 4^2} \approx 44.44$$

Worked again!

c) For set C:

We want a square number that is less than 33 so let's try 25.

$$q = 5$$

Check:

$$\frac{5^2 6^2}{6^2 - 5^2} \approx 81.82$$

Close enough!
No scary long-winded trial and errors needed although you could try that too. This method takes a bit of inspiration to come up with maybe but it's worth it in the end - converging functions are a powerful tool!

3.0.5 Qu17: Circular Geometry

a) Find the relationship between the grey area A_g and the area of the square A_s. Firstly they tell us that $A_g = f A_s$. The area of the square is simple:

$$A_s = (2xR)^2$$

The total white area is:

$$\frac{(2\pi - \theta)(xR)^2\pi}{2\pi} - \frac{(2\pi - \theta)R^2\pi}{2\pi} = \frac{2\pi - \theta}{2}[x^2 - 1]R^2$$

Therefore the grey area is given by the overall square - the white area:

$$(2xR)^2 - \frac{2\pi - \theta}{2}[x^2 - 1]R^2 = A_g = f(2xR)^2$$

$$\frac{4x^2(1-f)}{x^2 - 1} = \frac{2\pi - \theta}{2}$$

$$\theta = 2\pi - \frac{8x^2(1-f)}{x^2 - 1}$$

b) When $x = 3, f = 1/2$,

$$\theta = 2\pi - \frac{8(3^2)(1 - 1/2)}{3^2 - 1} = 2\pi - 9(1/2) = 1.7832$$

3.0.6 Qu18: Exponentials and Logarithms

This one is a pretty straightforward one: just expand, simplify and solve the resulting quadratic.

$$\frac{e^x + 9}{e^{-x} + 5} = 2$$

$$e^x + 9 = 2(e^{-x} + 5)$$
$$e^{2x} + 9e^x = 2 + 10e^x$$
$$e^{2x} - e^x - 2 = 0$$
$$(e^x - 2)(e^x + 1) = 0$$
$$e^x = 2, -1$$

For the second case, we get no solutions so our only solution for x comes from the first condition.

$$e^x = 2 \longrightarrow x = ln2$$

3.0.7 Qu19: Fireworks and Conservation Laws

a) Velocities of pieces after Explosion

We will use a combination of conservation of momentum and conservation of energy. First we define a sign convention; here we chose positive is upwards (vertical) and right (horizontal).

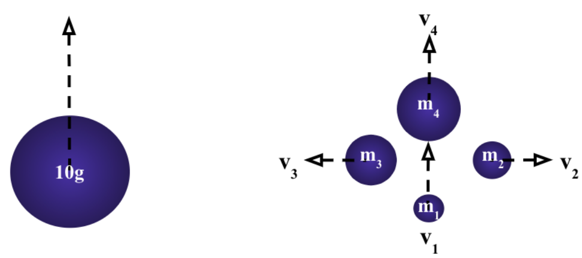

Figure 3: Qu19

Note: it doesn't matter what sign convention you use as long as you are consistent. The diagram also assigns directions to the masses for better visualisation but these are not taken into account in the working out e.g. masses 2 and 3 are shown to move in opposite directions in relation to each other, but in the calculations all velocities are added. This means that the answers should have a combination of +ve and -ve signs telling us the direction of the masses. If you decide to write out your calculations with the velocities as just magnitudes so that the directions in the diagram are the same as that indicated in your calculations, then you would expect your answers to all be +ve (or all -ve). Make sure whichever method you chose you were loyal to throughout your working out, otherwise you may get the wrong answer!

Conservation of Momentum (in the vertical direction)

$$m_4v_4 + m_1v_1 = 2 \times 10$$

$$4 + v_1 = 20$$
$$v_1 = 16ms^{-1}$$

Conservation of Momentum (in the horizontal direction)

$$m_2v_2 = -m_3v_3$$

$$2v_2 = -3v_3 \implies v_2 = -\frac{3}{2}v_3$$

Conservation of Energy

Total initial energy pre-explosion for a mass $M = m_1 + m_2 + m_3 + m_4 = 10g$. The total energy post-explosion converted to total KE of each of the pieces is the $KE_{initial} + E_{exp} = \frac{1}{2} \times 0.01 \times 2^2 + 1 = 1.02J$ or $1020gm^2s^{-2}$. We use the fact that the sum of the total initial energy is the sum of the individual KEs and combine with the relationship between v_2 and v_3 we found from conservation of momentum:

$$2040 = 16^2 + 2v_2^2 + \frac{4}{3}v_2^2$$

Solving we get $v_2 = 23.1ms^{-1}$ and $v_3 = -15.4ms^{-1}$.

b) The maximum speed occurs when mass m_1 moves up and the other masses move down. From conservation of momentum:

$$v_1 = -(2v_2 + 3v_3 + 4v_4)$$

3.0.8 Qu20: Interferometer

a) In our experience most of the students who got confused by this question got bogged down by the context. This part simply wants a sketch of the graph $I = I_p + I_q cos(kL)$ as a function of L which is essentially just an x,y-shifted cosine graph.

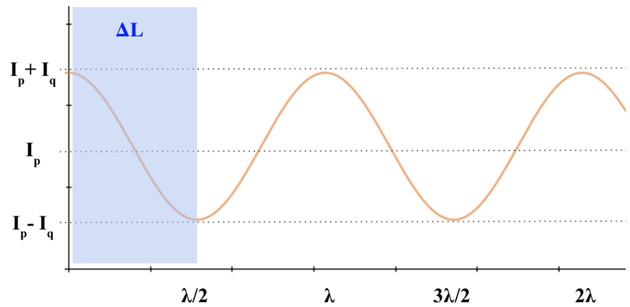

Figure 4: Qu20

b) To unambiguously decipher L means to be able to look at $I(L)$ and map it back definitively to one single value of L. This is only possible if we restrict the domain to $0 \leq L \leq \lambda/2$ as after this point each I can be mapped to multiple possible L and so the mapping is no longer unique.

3.0.9 Qu21: Electrical Switches

This is a bit of a confusing question, so if you found this tough to interpret, know you're not the only one!

When you have a long, wordy question like this, sometimes it is worth just "extracting" the important details before you get your hands dirty:

- n segments
- min delay L_{min}
- delay range ΔL
- delay l

From points 2 and 3, we can also deduce $L_{max} \leq L_{min} + \Delta L$.

a) Whenever the switch is in the lower position a delay l is active per segment. This would therefore lead to the overall minimum delay L_{min} for the entire series.

$$ln = L_{min}$$

$$l = \frac{L_{min}}{n}$$

b) Finding the delay of each segment L_i in terms of the resolution δL.

We have n segments in our system and each of these can either be switched "on" or "off" i.e. there are 2^n states the system may be found in corresponding to various permutations and combinations of ”on”s and ”off”s. The resolution δl has been introduced to make sure possible delays are evenly spaced out. If all switches are switched ”off” (lower) this is naturally the case as the delay l is constant. The issue appears when you have any other combination of ”on”s and ”off”s. Consider, we only have 1 switch in the system and it is switched ”off” (lower position) then the delay must be L_{min} for this route. However, instead say if the switch was ”on”, the delay applied would now be $L_{max} = L_{min} + \delta L$, as by definition we wanted the possible delays in the line to be ”increment-able” by multiples of δL between the minimum possible total delay and the maximum possible total delay. Every time we add 1 more segment into the sequence, we increase the number of states in the whole system by a factor of 2, i.e. $L_{max} = L_{min} + \delta L \times 2^{i-1}$. Choosing just the i^{th} segment in a line of n segments, we therefore find that the delay contribution is given by:

$$L_i = 2^{i-1}\delta L$$

If this is still confusing start by considering the delay of L_1, L_2, etc. We can then just extrapolate the pattern essentially.

The first section (on (1) or off (0)) defines the minimum possible resolution interval i.e. must be δL.

$$L_1 = \delta L$$

The second section now introduces 4 possible states (00, 01, 10, 11) so we should have 4 possible resolvable delay times: one is set by L_{min}, one by L_{max} so we need two more distinct possible delay values (than the two provided by the first section) allowed between the two (or two δLs should fit between the two max/min values).

$$L_2 = 2\delta L$$

By the third section we have 8 possibilities (000, 001, 011, 101, etc.), 4 more possibilities than the 2nd section allowed. To allow for 4 more possibilities, this 3rd section must contribute 4 more δLs to allow for the the distinguishing of these extra 4 possibilities as distinct delays.

$$L_3 = 4\delta L$$

The similar argument continues for the 4th section too. We have 16 possibilities total, 8 were accounted for by considering the 3 switch-system, and we have 8 more, so the 4th contribution is:

$$L_4 = 8\delta L$$

And so on for the 5th, 6th, 7th, etc. contributions. Extrapolating the results, you can seen that we end up with the same $L_i = 2^{i-1}\delta L$.

Two possible ways of thinking about it - hands down the question and the numerous variables in it does not make interpretation the simplest but this is the general gist.

c) The minimum necessary n can be found by considering the full range of L. From b) we had:

$$L_{max} - L_{min} = 2^{i-1}$$

and $\Delta L = L_{max} - L_{min}$. So the total:

$$\Delta L = \sum_{i=1}^{n} \delta L 2^{i-1}$$

This is just a geometric series. So applying the sum to n terms:

$$S_n = \frac{a(1-r^n}{1-r}$$

$$\Delta L = \delta L(2^n - 1)$$

Rearrange for n:

$$n = log_2(1 + \frac{\Delta L}{\delta L})$$

This is the minimum value of n required.

3.0.10 Qu22: Volume of Cone

Start by defining some variables:

- Full Depth: H
- Depth for half-max volume: h
- Radius of cone at max volume: D/2
- Radius of cone at half-max volume: r

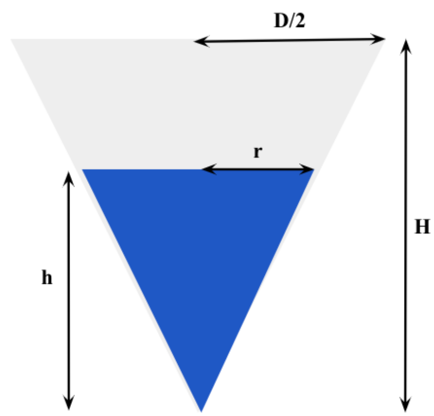

Figure 5: Qu22

We note immediately that from similar triangles (or geometries) we have:

$$\frac{H}{D} = \frac{h}{2r} \longrightarrow r = \frac{hD}{2H}$$

The general expression for the volume of a cone is $V = \frac{1}{3}\pi R^2 L$ for any base radius R and length of L. For the full max-volume case we have depth is H, volume is V and radius of base is $D/2$:

$$V = \frac{1}{3}\pi(\frac{D}{2})^2 H = \frac{\pi D^2 H}{12}$$

For $V_1 = V/2$, the half-max volume case we first consider the expression we get from considering the volume of a cone and the relation $r = \frac{hD}{2H}$:

$$V_1 = \frac{1}{3}\pi r^2 h = \frac{\pi h}{3}\frac{h^2 D^2}{4H^2} = \frac{\pi h^3 D^2}{12H^2}$$

Applying now that $V_1 = V/2$ using our previous expression for V and solving for h:

$$\frac{\pi h^3 D^2}{12H^2} = \frac{\pi D^2 H}{24}$$

$$h = \frac{H}{2^{1/3}}$$

3.0.11 Qu23: Geometric Series

With each pass $1/n$th of the total amount of impurities remaining is removed. Therefore if we indefinitely continued our filtration process, after n passes, the total fraction removed is given by:

$$\frac{1}{n} + (\frac{1}{n})^2 + ... + (\frac{1}{n})^a$$

as $a \longrightarrow \infty$, so we basically want the sum to infinity of the geometric series $F = Ar^{i-1}$ where $A = 1/n$ and $r = 1/n$.

$$F_\infty = \frac{A}{1-r} = \frac{1/n}{1-\frac{1}{n}} = \frac{1}{n-1}$$

When $n = 2 \implies F_\infty = 1$. So all of the impurities can get removed.

However, when $n = 3 \implies F_\infty = 1/2$ so the maximum fraction of impurity that can be removed is 1/2.

3.0.12 Qu24: Springs

a) The Equilibrium Extension

The total length of the spring is the sum of its natural length and extension:

$$R_T = R_0 + R$$

We can use a combination of Hooke's law and Newton's second law (for constant mass) to find an expression for the equilibrium extension.

$$F = m\omega^2 R_T = kR$$

$$m\omega^2 R_0 = R(k - m\omega^2)$$

$$R = \frac{m\omega^2 R_0}{k - m\omega^2}$$

b) Breaking Frequency

Spring breaks at

$$F_{max} = kR_{max} = k\frac{m\omega_c^2 R_0}{k - m\omega_c^2}$$

Rearranging for ω_c:

$$\omega_c = (\frac{kF_{max}}{m(kR_0 + F_{max})})^{1/2}$$

Note that the R_{max} was a maximum *extension* taken from the expression in part a).

c) Sketching

For the numerical values given our equation reduces to the following form:

$$\omega_c = \sqrt{\frac{F_{max}}{1 + F_{max}}}$$

This graph would typically behave like the $x^{1/2}$ graph at first order for small values of F_{max} (you can check this by applying a quick Taylor expansion). For large values of F_{max} it will tend to 1, so we end up with a plateauing form. However, we only want the domain between $0 \leq F_{max} \leq 1$N for which the maximum $\omega_c = \frac{1}{2}$.

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