Physics Aptitude Test (PAT) Unofficial Solutions: 2016

University of Oxford Admissions Test

Physics, Engineering, Materials Science

Solutions by PhysOx Initiative
2020

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1 Foreword

The solutions provided here are by no means an official set of answers; our aim was to provide unofficial solutions in order to provide those studying for the PAT with the means to check their answers as they attempt past paper questions. There is no mark breakdown but we aimed to provide detailed explanations on how to solve the questions and develop a good intuition for them. These solutions have been compiled by our team of Oxford Physics graduates who have all taken the PAT and our team members also have experience in marking the PAT and/or running the PAT summer programmes coordinated by the University of Oxford.

Some general tips for questions:

1) Try and **keep your calculations in terms of symbols and letters** until you see your calculations simplifying considerably by substituting numbers. This will genuinely reduce errors in your work and make it way easier for the marker to understand your thought process.

2) Give your **variables reasonable names**, for example don't call your initial velocity something like v_u and final velocity something like v_v, as you will definitely confuse the life out of everyone looking at your work not to mention yourself.

3) Your teachers may say this a lot and many of you probably ignore it but **drawing diagrams really helps!** Sometimes the best way to deal with paragraphs of information is a simple drawing which has all the important bits - it will also save you a lot of time!

4) Always **show your working** so it does not look like you picked an answer out of thin air especially for the longer answer questions so it's easier to pick up on exactly where you went wrong if you do go wrong. Fair enough you may have had a moment of next level inspiration but a couple of lines (or words here and there) just outlining your way of thinking really helps. That said, it doesn't have to be an essay!

5) Don't feel the need to rush, **relax** yourself and approach the questions. If you get really stuck on one, don't get too put off, just skip it and come back to it later if you get time.

2 Section A: Mathematics for Physics

2.1 Qu1: Differentiation

A combination of Chain rule and Product rule.

$$y = xsin(x^2)$$
$$\frac{dy}{dx} = sin(x^2) + 2x^2cos(x^2)$$

2.2 Qu2: Trigonometric Equations

Just a quadratic equation with $tan\theta$. Factorise and solve as usual.

$$\sqrt{3}tan^2\theta - 2tan\theta - \sqrt{3} = 0$$
$$\sqrt{3}tan^2\theta - 3tan\theta + tan\theta - \sqrt{3} = 0$$
$$\sqrt{3}tan\theta(tan\theta - \sqrt{3}) + (tan\theta - \sqrt{3}) = 0$$
$$(\sqrt{3}tan\theta + 1)(tan\theta - \sqrt{3}) = 0$$
$$tan\theta = \frac{-1}{\sqrt{3}}, \sqrt{3}$$
$$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{3}, \frac{4\pi}{3}$$

2.3 Qu3: Logarithms and Simultaneous Equations

First we should go over some rules for logs.

$$log_a b = c \implies a^c = b \quad (1)$$
$$log_a a = 1 \quad (2)$$
$$log_a b^d = c \implies log_a b = \frac{c}{d} \quad (3)$$
$$log(\frac{a}{b}) = loga - logb \quad (4)$$

Now let's simplify the two equations and see what we get:

$$log_4(\frac{64^x}{16^y}) = 13$$
$$log_4 64^x - log_4 16^y = 13$$
$$3x - 2y = 13 \quad (5)$$

The second equation can be simplified more easily:

$$log_{10}10^x + log_3 3^y = 1$$

$$x + y = 1 \tag{6}$$

Now solving the two equations:

$$3x - 2y = 13$$
$$-(2x + 2y = 2)$$
$$5x = 15 \implies x = 3$$
$$y = -2$$

2.4 Qu4: Binomial Expansions

We need to find the combination which gives us x^0 before we find the coefficient from the binomial expansion. We want:

$$x^k \times \frac{1}{x^{2(12-k)}} = x^0$$
$$x^k(x^{2k-24}) = x^0$$
$$3k - 24 = 0$$
$$k = 8$$

This is what we substitute into our binomial coefficient equation:

$$\binom{n}{k} = {}^nC_k = \frac{n!}{k!(n-k)!} \tag{7}$$

Now $n = 12$ and $r = 8$ as we found:

$${}^{12}C_8 = \frac{12!}{8!4!} = 495$$

2.5 Qu5: Combinations

We have access to 5 numbers and we are told we need to make a number greater than 5000. There are two possibilities to consider:

a) We have *5 numbers* to choose from.

In this case we are allowed any choice of numbers for the 5 digits as long as we remember no repetitions:

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$$

So in this scenario we get 120 possible numbers greater than 5000.

b) We have *4 numbers* to choose from.

Here we need to be more careful. The first digit can only be $5, 6 or 7$ but after than we can choose any of the 4 remaining digits to fill up the 3 remaining digits as long as no repeats.

$$3 \times 4 \times 3 \times 2 = 72$$

So for 4 digits we have 72 combinations.

So the overall is now $72 + 120 = 192$.

2.6 Qu6: Geometric Progression

Each twig has two leaves and each month the number of twigs double each with two leaves - smells geometric progression-y!

The first term $a = 2$.
The common ratio $r = 2$.
We want the sum to 10 terms.

$$S_{10} = \frac{a(r^{10} - 1)}{r - 1} = \frac{2(2^{10} - 1)}{1}$$

$$S_{10} = 2046$$

2.7 Qu7: 6-sided Die and Probabilities

The exact order is: 6 5 4 3 3 3 3 2 2 1.
That is a minimum of 10 throws.

a) We need a minimum of 10 throws so there's no chance of winning with 8 - Probability = 0.

b) Probability of rolling any one number is $1/6$ so the total probability is $(\frac{1}{6})^{10}$.

c) We can either start from the $1st$ throw, $2nd$ throw or $3rd$ throw. So the probability is:

$$3 \times (\frac{1}{6})^{10} = \frac{3}{6^{10}}$$

2.8 Qu8: Areas

Let's start with the easy part:

$$A_{circ} = \pi r^2$$

Now we consider the area of the octagon which is the area of a square − the area of the four triangles.

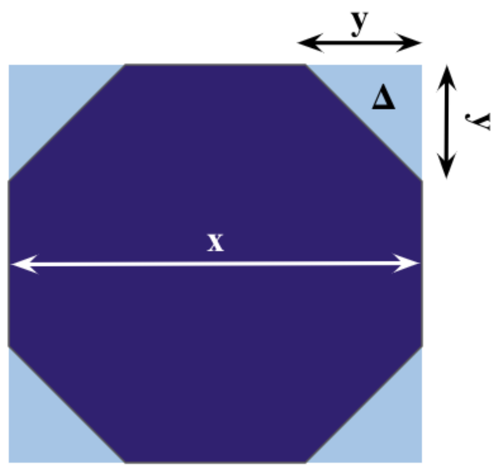

Figure 1: Qu8

$$A_{oct} = x^2 - 4\Delta$$

Where the area of one triangle corner is Δ.

$$\Delta = \frac{1}{2}y^2$$

From Pythagoras:

$$z^2 = 2y^2$$

$$\Delta = \frac{z^2}{4}$$

$$A_{oct} = x^2 - z^2$$

And now we need to get rid of our z dependence using the fact that looking at one side of the octagon $x = z + 2y = z(1 + \sqrt{2})$.

$$A_{oct} = \frac{2x^2}{1+\sqrt{2}} = A_{circ} = \pi r^2$$

$$x = (\frac{\pi r^2(1+\sqrt{2})}{2})^{\frac{1}{2}}$$

2.9 Qu9: Inequalities

$$5 - 3x < \frac{2}{x}$$

Multiply by x^2:

$$x(3x^2 - 5x + 2) > 0$$
$$x(3x^2 - 3x + 2x + 2) > 0$$
$$x(3x(x-1) - 2(x-1)) > 0$$
$$x(3x-2)(x-1) > 0$$

Solutions are:

$$0 < x < \frac{2}{3}$$
$$x > 1$$

2.10 Qu10: Area under Curves

First we really should sketch this out.

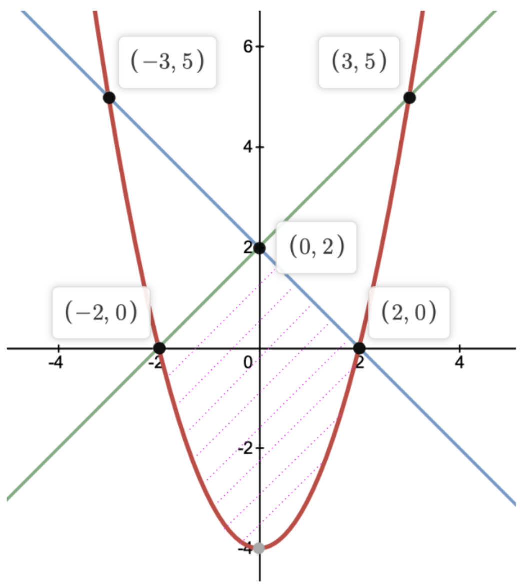

Figure 2: Qu10

Area of the triangle $A_1 = \frac{1}{2} \times 2 \times 4 = 4$.

The area between the quadratic to the y axis.

$$2\int_0^2 x^2 - 4dx = 2[\frac{x^3}{3} - 4x]_0^2 = 2[\frac{8}{3} - 8] = \frac{-32}{3}$$

This is negative as we expect but we want the overall area:

$$Total = 4 + \frac{32}{3} = \frac{44}{3}$$

2.11 Qu11: Chain Rule

The rate of the height decrease:

$$h(t) = h_0 - \alpha t$$

$$\frac{dh}{dt} = -\alpha$$

The volume of a cylinder:

$$V = \pi r^2 h \tag{8}$$

So $h = \frac{V}{\pi} r^{-2}$.

$$\frac{dh}{dr} = \frac{-2V}{\pi} r^{-3}$$

$$\frac{dr}{dt} = \frac{dh}{dt}\frac{dr}{dh} = \frac{\alpha\pi}{2V} r^3 = \frac{\alpha\pi}{2V}(\frac{V}{\pi h})^{\frac{3}{2}}$$

All that's left to do is substitute:

$$h(t) = h_0 - \alpha t$$

Therefore as time increases, the rate increases (as the denominator decreases).

3 Section B: Physics

3.1 Qu12: A Ball Bouncing

We assume the main energy transfer is from GPE to kinetic energy and vice versa other than the KE loss you get during the bounce which accounts for the friction and non-elasticity of the ball-surface interaction.

We start off with $GPE = mgh = mg$ as the ball starts off at a height $h_0 = 1m$.

After each bounce we are told the ball loses $\frac{1}{4}$ of its energy. This means that it retains $\frac{3}{4} mgh_{previous}$.

After 1 bounce:

$$mgh_1 = (\frac{3}{4}) mgh_0$$

After n bounces:

$$h_n = (\frac{3}{4})^n$$
$$(\frac{3}{4})^n < \frac{1}{4}$$
$$nln(\frac{3}{4}) > ln(\frac{1}{4})$$
$$n > 4.81$$

So we need 5 bounces.

3.2 Qu13: Resistant-Temperature Characteristics

a) An ideal wire is a perfect conductor therefore should have zero resistance and that applies across all temperatures so should also apply around room temperature.

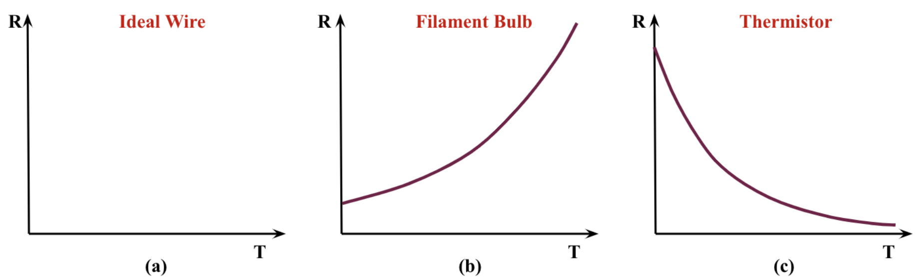

Figure 3: Qu13

b) For a filament light bulb as the temperature increases, from kinetic theory, we expect the resistance to also increase.

c) The thermistors we usually deal with are NTC thermistors so the resistance decreases with temperature.

3.3 Qu14: Kepler's Laws and Moons

We have circular orbits and need to find the ratio of orbital periods so definitely need to be looking at Kepler's relation. Starting from the derivation by setting the gravitational force of attraction equal to the centripetal force:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \tag{9}$$

For circular orbits we also have that

$$v = \frac{2\pi r}{T} \tag{10}$$

for T the time period.

$$\frac{(2\pi)^2 r^2}{T^2} = \frac{GM}{r}$$

$$T^2 \propto r^3$$

So we know that $\frac{T^2}{r^3} = const$. We can use this to find the ratio:

$$\frac{T_E^2}{r_E^3} = \frac{T_I^2}{r_I^3}$$

$$(\frac{r_E}{r_I})^{\frac{3}{2}} = \frac{T_E}{T_I}$$

$$\frac{T_E}{T_I} = 2.02$$

3.4 Qu15: Rower on River

Let's write down what we know:
(1) Distance to cover is $100m$. This is the shortest distance between the two banks.

(2) We have $10s$ to cross the river.

(3) In that $10s$ the river will push the boat at a rate $7.5ms^{-1}$ horizontally across the river. The distance the boat is pushed horizontally should be $75m$. So when the rower crosses the river she must aim to reach the other side $75m$ behind her initial position in order to reach directly opposite to her initial position.

(4) The distance she must travel is given by $s = 10v$ as she has $10s$ to cross.

Putting all of this together in a diagram:

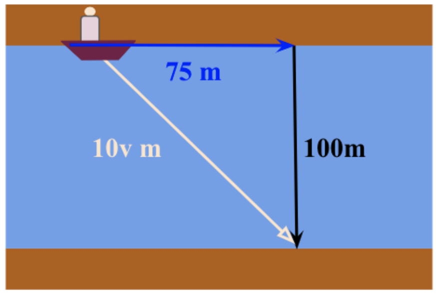

Figure 4: Qu15

$$10v = \sqrt{75^2 + 100^2}$$
$$v = 12.5ms^{-1}$$
$$tan\theta = \frac{75}{100} = \frac{3}{4}$$

Therefore from the diagram, relative to the water the angle that she must row is given by:

$$\phi = \frac{\pi}{2} + arctan\frac{3}{4}$$

3.5 Qu16: Coulomb's Law and Equilibrium

For equilibrium we need the net force on the point charge to be zero.

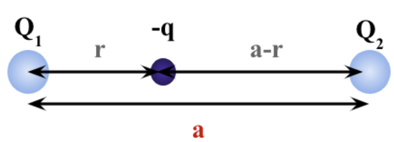

Figure 5: Qu16

Considering the contributions of the two charges at the point of the negative charge:

$$\frac{Q_1}{r^2} = \frac{Q_2}{(a-r)^2} \tag{11}$$

a) When $Q_1 = Q_2$

$$(a-r)^2 = r^2$$
$$r = \frac{a}{2}$$

a) When $Q_1 \neq Q_2$

From the equation 11 for the general equilibrium condition:

$$Q_1(a-r)^2 = Q_2 r^2$$

$$r^2(Q_1 - Q_2) - 2arQ_1 + a^2 Q_1 = 0$$

$$r = \frac{aQ_1 \pm \sqrt{a^2 Q_1^2 - a^2 Q_1 (Q_1 - Q_2}}{(Q_1 - Q_2}$$

$$r = \frac{a(1 \pm \sqrt{\frac{Q_2}{Q_1}})}{1 - \frac{Q_2}{Q_1}}$$

$$r = \frac{a}{1 \pm \sqrt{\frac{Q_2}{Q_1}}}$$

We want the solution:

$$r = \frac{a}{1 + \sqrt{\frac{Q_2}{Q_1}}}$$

As then the point lies between the two charges and the unused solution lies outside the line between the two charges. This is possible when the two charges have opposite charges. The used solution therefore applies for like charges and the unused for unlike charges.

3.6 Qu17: Force-Displacement Graphs

a) Work done = area under F-s graph.

$$WD = \int F ds = \frac{1}{2}(5 \times 10) = 25Ncm = 0.25J$$

b) $KE = \frac{1}{2}mv^2 = 0.25J$.
Rearrange for v:

$$v = 5ms^{-1}$$

c) First thing to do is write down the relationship between force and displacement.

$$F = -ks$$

Note that s is displacement. This is hooke's law so we basically have a spring. Now from Newton's Second Law in the special case of constant mass, we recover $F = ma = m\frac{d^2s}{dt^2}$:

$$m\frac{d^2s}{dt^2} = -ks$$

Which is a standard simple harmonic motion differential equation with solution:

$$s = Asin(\sqrt{\frac{k}{m}}t)$$

You can stick this back into the to convince yourself that this is indeed a legitimate solution. So we expect an oscillatory behaviour characterised by some amplitude and period.

$$\sqrt{\frac{k}{m}}T = 2\pi$$

where k is the (-) gradient $2Ncm^{-1} = 200Nm^{-1}$.

$$T = \frac{2\pi}{(\frac{200}{0.02})^{0.5}}$$

$$T = \frac{\pi}{50} = 0.063s$$

This is the period of the oscillation. The amplitude is the maximum displacement, so the displacement the mass was dropped from, which the question told us: $5cm$.

3.7 Qu18: Dimensional Analysis

$$F = kr^x\eta^y v^z \tag{12}$$

$$kgms^{-2} = m^x kg^y m^{-y} s^{-y} m^z s^{-z}$$
$$= m^{x-y+z} kg^y s^{-z-y}$$

Now matching up coefficients:

y = 1

Then:

$$-z - 1 = -2$$

z=1

$$x - 1 + 1 = 1$$

x=1

3.8 Qu19: Photoelectric Effect

It's a good habit to make notes on wordy questions so you know what useful information you have:

$$\lambda = 625 \times 10^{-9}m$$

$$\phi = 1.0eV$$

a) Maximum Speed

Incident photon energy - Work function energy = Kinetic Energy (KE)

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$$

$$v = [\frac{2}{m}(\frac{hc}{\lambda} - \phi)]^{\frac{1}{2}} = [\frac{2}{1 \times 10^{-30}}(\frac{6 \times 10^{-34} \times 3 \times 10^{8}}{625 \times 10^{-}9} - 1.6 \times 10^{-}19)]^{0.5}$$

$$v = 5.1 \times 10^{5} ms^{-1}$$

It is also good practice to keep things in terms of symbols until right at the end unless the numbers are nice and your equations simplify down better on substitution beforehand.

b) $\Delta KE = 5 \times 10^{3} eV$.

$$= 1.6 \times 10^{-19} \times 5 \times 10^{3}$$

$$\Delta KE = 8 \times 10^{-16} J$$

$$\frac{1}{2}mv_f^2 = KE_{initial} + \Delta KE$$

$$= (\frac{hc}{\lambda} - \phi) + (8 \times 10^{-16})$$

$$v_f = 4 \times 10^{7} ms^{-1}$$

3.9 Qu20: Circuits and Specific Heat Capacity

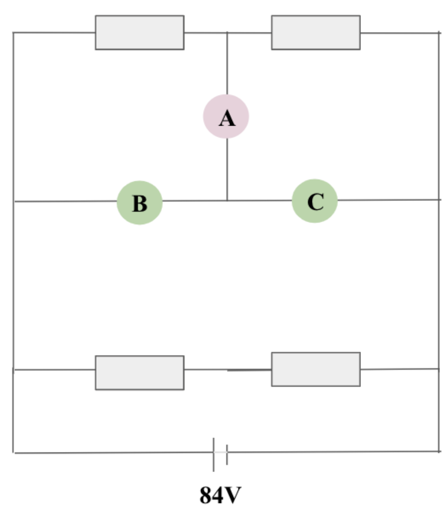

Figure 6: Qu20

Current through each branch is the same so by symmetry we should have

$$V_B = V_C$$

Similarly if we look at the voltage at the nodes either side of A, they fall in parallel branches to the applied voltage. Voltages in parallel are the same so the voltage at both nodes are the same and the p.d. across A should be 0.

$$V_A = 0$$

Now we know that power (P) is rate of change of energy (Q):

$$P = \frac{Q}{t} \implies Q = Pt$$

And our famous heat capacity formula has to also be incorporated:

$$Q = mc\Delta T$$

Combining:

$$Pt = mc\Delta T$$

$$\frac{V^2}{R}t = mc\Delta T$$

$$t_C = t_B = \frac{mc\Delta R}{V^2} = \frac{(27-20)6 \times 4200}{42^2} = 100s$$

So for B and C it will take $100s$. For A, no power dissipated due to the voltage being 0 so:

$$t_A = \infty$$

3.10 Qu21: Refraction through a Sphere

a) Refractive index of vacuum = 1.
Snell's Law:

$$nsin\theta_1 = 1sin45$$

From the diagram, $sin\theta_1 = \frac{1}{2}$.

$$\frac{n}{2} = \frac{1}{\sqrt{2}}$$

$$n = \sqrt{2}$$

b) Need critical angle (for total reflection) $\implies \theta_2 = \pi/2$.

$$sin\theta_1 = \frac{n_2}{n_1} = \frac{1}{n}$$

$$sin\theta_1 = \frac{1}{\sqrt{2}} \implies \theta_1 = 45\,\text{deg}$$

So the condition for the complete reflection of the beam is satisfied for any angle

$$\theta_1 > 45\,\text{deg}$$

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