Physics Aptitude Test (PAT) Unofficial Solutions: 2015

University of Oxford Admissions Test

Physics, Engineering, Materials Science

Solutions by PhysOx Initiative
2020

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Contents

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1 Foreword

The solutions provided here are by no means an official set of answers; our aim was to provide unofficial solutions in order to provide those studying for the PAT with the means to check their answers as they attempt past paper questions. There is no mark breakdown but we aimed to provide detailed explanations on how to solve the questions and develop a good intuition for them. These solutions have been compiled by our team of Oxford Physics graduates who have all taken the PAT and our team members also have experience in marking the PAT and/or running the PAT summer programmes coordinated by the University of Oxford.

Some general tips for questions:

1) Try and **keep your calculations in terms of symbols and letters** until you see your calculations simplifying considerably by substituting numbers. This will genuinely reduce errors in your work and make it way easier for the marker to understand your thought process.

2) Give your **variables reasonable names**, for example don't call your initial velocity something like v_u and final velocity something like v_v, as you will definitely confuse the life out of everyone looking at your work not to mention yourself.

3) Your teachers may say this a lot and many of you probably ignore it but **drawing diagrams really helps!** Sometimes the best way to deal with paragraphs of information is a simple drawing which has all the important bits - it will also save you a lot of time!

4) Always **show your working** so it does not look like you picked an answer out of thin air especially for the longer answer questions so it's easier to pick up on exactly where you went wrong if you do go wrong. Fair enough you may have had a moment of next level inspiration but a couple of lines (or words here and there) just outlining your way of thinking really helps. That said, it doesn't have to be an essay!

5) Don't feel the need to rush, **relax** yourself and approach the questions. If you get really stuck on one, don't get too put off, just skip it and come back to it later if you get time.

2 Section A: Mathematics for Physics

2.1 Qu1: Binomial Expansion

Standard binomial expansion:

$$\binom{n}{k} = {}^nC_k = \frac{n!}{k!(n-k)!} \tag{1}$$

We can now expand the terms:

$$(2x + x^2)^5$$

$$= (2x)^5 + {}^5C_4(2x)^4(x^2) + {}^5C_3(2x)^3(x^2)^2 + {}^5C_2(2x)^2(x^2)^3 + {}^5C_1(2x)^1(x^2)^4 + (x^2)^5$$

$$= 32x^5 + 80x^6 + 80x^7 + 40x^8 + 10x^9 + x^{10}$$

2.2 Qu2: Logarithm

Rules to remember:

$$log_a b = c \implies a^c = b \tag{2}$$

$$a^{log_a b} = b \tag{3}$$

Now simplifying the equation:

$$log_2 x + log_4 16 = 2$$

$$log_2 x + 2 = 2$$

$$log_2 x = 0 \implies 2^{log_2 x} = 2^0$$

$$x = 2^0 = 1$$

2.3 Qu3: Geometric Series

a) Sum to n terms: First term $a = \frac{1}{3}$.
Common Ratio $r = \frac{1}{3}$.

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{\frac{1}{3}}{\frac{2}{3}}(1 - (\frac{1}{3})^5) = \frac{121}{243}$$

b) Sum to ∞:

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

2.4 Qu4: Integration by Substitution

Usually with integration you have 3 options:

1. Simple integral - you just need to identify the function.

2. By Parts - usually a product of a few simple-ish functions that simplify on differentiation of one part.

3. By Substitution - when you have more than one function applied to a function - as in this case!

Now it looks a bit complex so let's expand a bit and see what we get, ideally we want to be able to cancel something to simplify it.

$$\int_4^6 [(2x-6)][(x^2-6x+8)]^{\frac{1}{2}} dx$$

Now if I substitute $u = x^2 - 6x + 8$ then we also have $\frac{du}{dx} = 2x - 6$.

$$\int_0^8 [u^{\frac{1}{2}}] du$$

$$[\frac{2u^{\frac{3}{2}}}{3}]_0^8 = \frac{2}{3}(\sqrt{8})^3 = \frac{32\sqrt{2}}{3}$$

2.5 Qu5: Quadratics

For no real solutions of a quadratic $ax^2 + bx + c = 0$ we want $b^2 - 4ac < 0$.

Let's rearrange:

$$4x^2 + 8x - 8 = 4mx - 3m$$
$$4x^2 + (8-4m)x + (3m-8) = 0$$
$$(8-4m)^2 - 16(3m-8) < 0$$
$$(2-m)^2 - (3m-8) < 0$$

Expand and simplify:

$$m^2 - 4m + 4 - 3m + 8 < 0$$
$$m^2 - 7m + 12 < 0$$
$$(m-4)(m-3) < 0$$
$$3 < m < 4$$

2.6 Qu6: Coins and Probabilities

The coin is tossed 3 times $n = 3$.

$$p(head) = p(tail) = 0.5$$

a) Careful: says 2 or more *in succession*.
So we can have: TTT, HTT or TTH but not THT.

$$3 \times (\frac{1}{2})^3 = \frac{3}{8}$$

b) 2 consecutive tosses that are the same?

TTH, HTT, HHT, THH $\implies 4 \times (\frac{1}{2})^3 = \frac{1}{2}$
c) If any one toss is tails then all tosses are tails:

$$P(anyoneistails) = 1 - P(allheads) = 1 - (\frac{1}{2})^3 = \frac{7}{8}$$

$$P(alltails) = (\frac{1}{2})^3 = \frac{1}{8}$$

Then:

$$\frac{P(alltailsANDonetail)}{P(onetailknown)} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{1}{7}$$

2.7 Geometry and Trigonometry

Area of small circle πr^2.
Area of big circle πR^2.

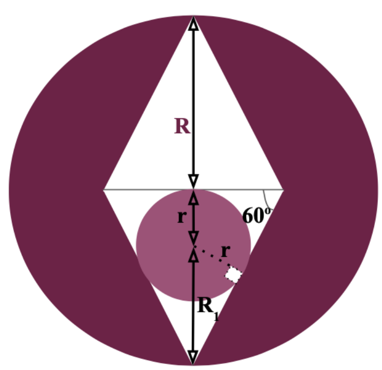

Figure 1: Qu7

Now we need the area of the triangles:

$$tan60 = \frac{R}{x}$$

$$x = \frac{R}{tan60} = \frac{R}{\sqrt{3}}$$

Area of triangle $= \frac{R}{\sqrt{3}} \times R = \frac{R^2}{\sqrt{3}}$.

Relationship between the radius of the smaller circle r and R_1.

$$cos60 = \frac{r}{R_1} \implies R_1 = 2r$$

$$R = 3r$$

So shaded areas $= \pi R^2 - \frac{2R^2}{\sqrt{3}} + \pi r^2$.

We want this in terms of r:

$$10\pi r^2 - \frac{18r^2}{\sqrt{3}} = r^2(10\pi - 6\sqrt{3})$$

Area is then $2r^2(5\pi - 3\sqrt{3})$.

2.8 Tangents and Normals

$(x+3)^2 + (y-3)^2 = 17$ To find the gradient, differentiate:

$$2(x+3) + 2(y-3)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x+3}{3-y}$$

At point $(1, 2)$:

$$\frac{dy}{dx} = \frac{4}{1} = 4$$

Tangent $m = 4$ and normal $m = -\frac{1}{4}$ as tangent $\times$ normal $= -1$.

Tangent

$$y = mx + c$$

$$y = 4x + c$$

$$2 = 4 + c$$

$$c = -2$$

$$y = 4x - 2$$

This is the equation of the tangent.

Normal

$$y = mx + c$$
$$y = \frac{-1}{4}x + c$$
$$2 = \frac{-1}{4} + c$$
$$c = \frac{9}{4}$$
$$4y = -x + 9$$

2.9 Graph Sketching

Look at Asymptotes

when $x^2 = 4$ and denominator $= 0$ so $x = \pm 2$ will give asymptotes.

Turning Points

$$y = 8(4 - x^2)^{-1} - 3$$
$$\frac{dy}{dx} = 16x(4 - x^2)^{-2}$$

so $x = 0$ and $y = -1$.

Limits

$$x \to \infty, y \to -3$$
$$x \to -\infty, y \to -3$$

Intercepts

$$x = 0, y = -1$$

The turning point.

$$y = 0, \frac{8}{4 - x^2} = 3 \implies 8 = 12 - 3x^2$$
$$3x^2 = 4$$
$$x = \pm \frac{2}{\sqrt{3}}$$

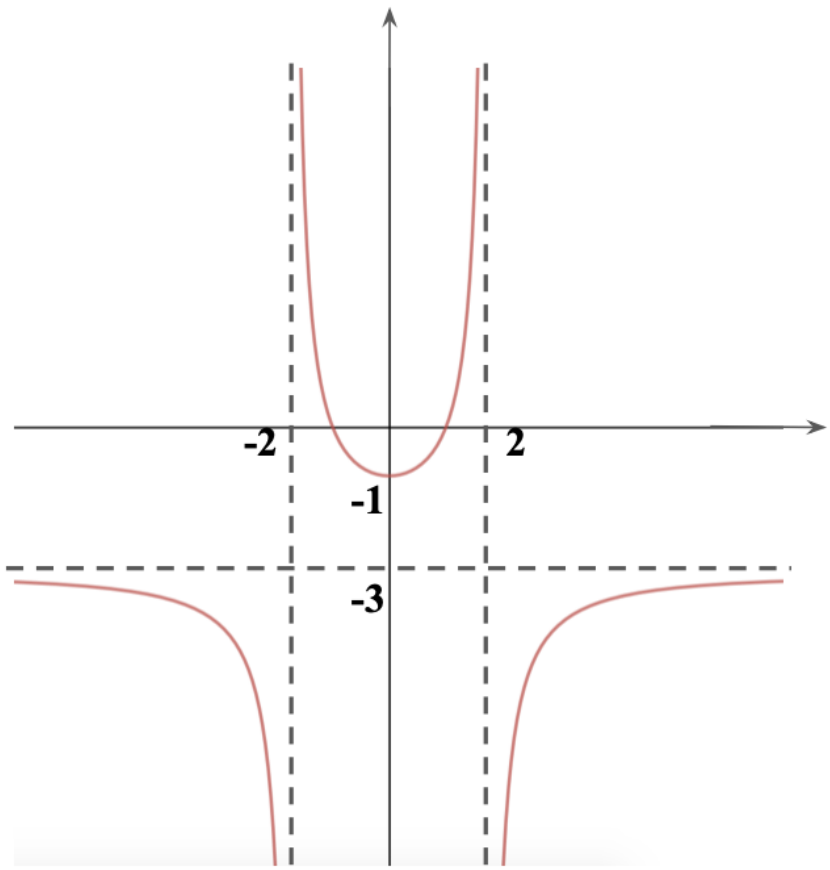

Figure 2: Qu9

The range of the function is thus:

$$y \geq 1 and y < -3$$

2.10 Inequalities

$$-1 < \frac{3x+4}{x-6} < 1$$

Separate to the two cases:

1. Case 1

$$\frac{3x+4}{x-6} < 1$$
$$(3x+4)(x-6) < (x-6)^2$$

$$(x-6)[3x+4-x+6] < 0$$
$$(x-6)(2x-10) < 0$$
$$-5 < x < 6$$

2. Case 2

$$-1 < \frac{3x+4}{x-6}$$
$$-(x-6)^2 < (3x+4)(x-6)$$
$$0 < (x-6)[3x+4+x-6]$$
$$(x-6)(4x-2) > 0$$
$$x < \frac{1}{2}$$

or

$$x > 6$$

So overall

$$-5 < x < \frac{1}{2}$$

3 Section B: Physics

3.1 Qu11: Projectiles

This question just requires knowing the SUVAT equations and being observant.

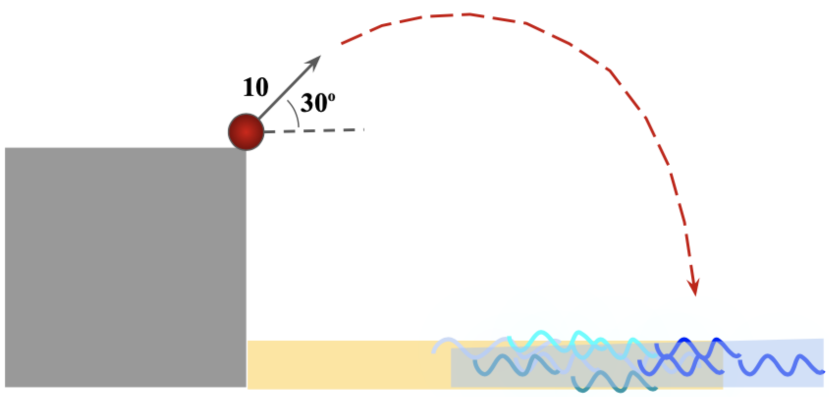

Figure 3: Qu11

Considering the vertical direction:

$$s = -10$$

$$u = 10sin30 = 5$$
$$v = ?$$
$$a = -10$$
$$t =$$

The thing to note is that $s = -10m$ as we are looking for the time to a displacement that is $10m$ below the top of the cliff. The equation we need is:

$$s = ut + \frac{1}{2}at^2$$
$$-10 = 5t - 5t^2$$
$$t^2 - t - 2 = 0$$
$$(t-2)(t+1) = 0$$
$$t = -1, 2$$

The negative solution comes from extrapolating the quadratic back to the ground which you can see would happen in ”negative” time (i.e. before the projectile is launched from a point 10m above). The time to hit the beach below is therefore the positive solution:

$$t = 2s$$

3.2 Qu12: Energy Conservation and Forces

We assume no friction.

Now if the distance half way down the slope is l then considering the conservation of energy at that point:

$$\frac{1}{2}mv^2 = mglsin(\alpha)$$
$$v = (2glsin\alpha)^{\frac{1}{2}}$$

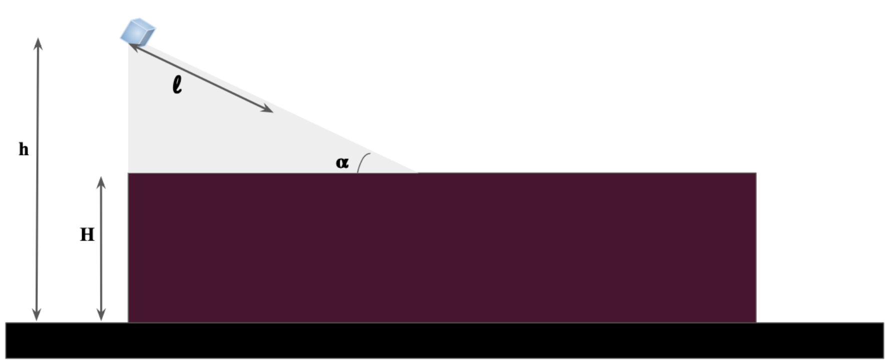

Figure 4: Qu12

Now we need an expression for $lsin\alpha$:

$$2lsin\alpha = h - H$$
$$lsin\alpha = \frac{h - H}{2}$$
$$v = [g(h - H)]^{\frac{1}{2}}$$

3.3 Qu13: Solar Eclipses

This is primarily a geometric problem and we just have to look at similar triangles. For the moon to block the Sun, the furthest transverse point from the axis of alignment of the the three bodies, of both the Moon and the Sun (i.e. points one moon-radius/ one Sun-radius from the axis) should lie on the same line. This is shown, perhaps less confusingly than the worded version offered above, in the following diagram.

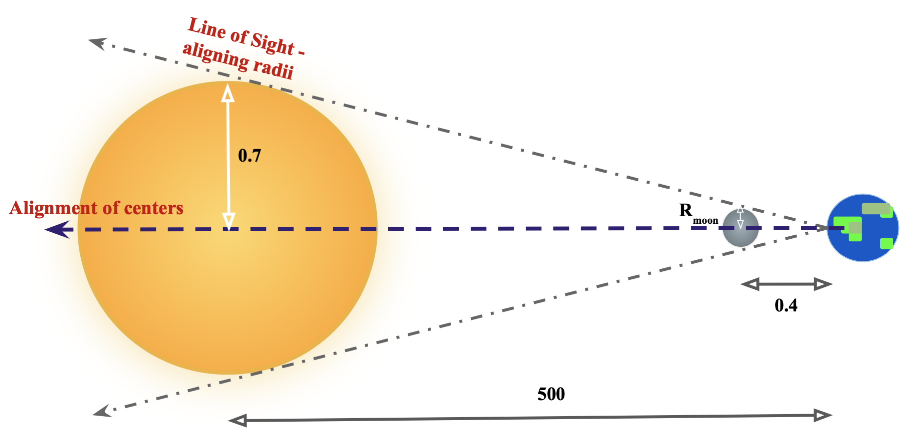

Figure 5: Qu13

Note as we will have to deal with ratios the units of the length scales do not matter too much so long as they are consistent. The next part is just a bit of geometry.

$$\frac{R}{0.4} = \frac{0.7}{150}$$
$$R_{moon} = \frac{0.28}{150} = 1.87 \times 10^{-3} \times 10^{6}$$
$$R_{moon} = 1.87 \times 10^{3} km$$

3.4 Qu14: Snell's Law and Refraction

a) Start with Snell's Law.

$$n_1 sin\theta_1 = n_2 sin\theta_2 \tag{4}$$

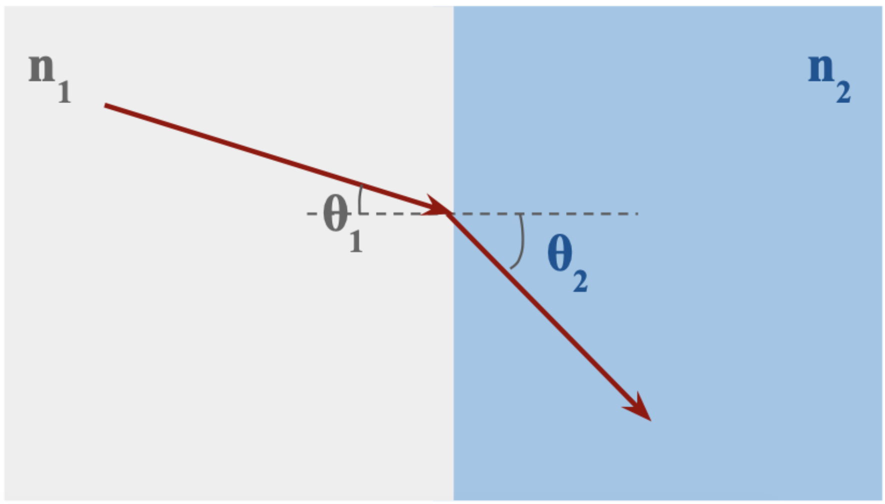

Figure 6: Qu14

b) Total internal reflection will occur when for $n_2 < n_1$, $\theta_1 > \theta_{crit}$. This θ_{crit} occurs when $\theta_2 = 90$.

$$n_1 sin\theta_{crit} = n_2 sin 90$$
$$\theta_{crit} = arcsin(\frac{n_2}{n_1})$$

3.5 Qu15: Forces and Equilibrium

We want to resolve these forces, so we can start by balancing in the "horizontal" direction:

$$F_A = F_C cos 45$$
$$F_A = F_C / \sqrt{2}$$
$$\sqrt{2} F_A = F_C$$

Now considering vertical balance:

$$F_B = F_C cos 45$$

So

$$F_A = F_B$$

3.6 Qu16: Elastic Collisions

First, a quick diagram. Remember to be consistent with directions and signs - it doesn't matter whether left is +ve and right is -ve or the other way around, make sure you stick to the same convention!

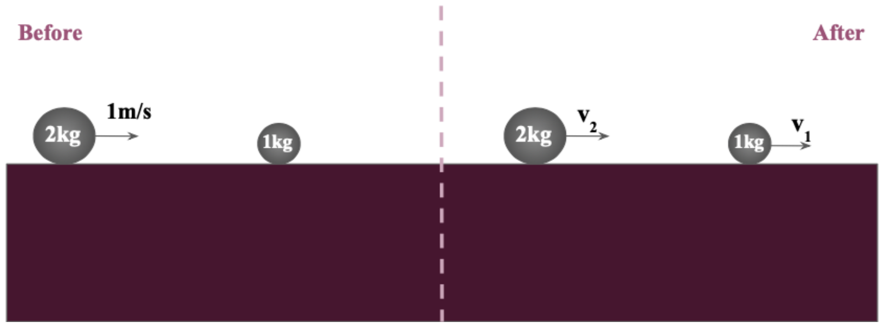

Figure 7: Qu16

For elastic collisions we have conservation of momentum and kinetic energy to consider.

Conservation of momentum:

$$2 = 2v_2 + v_1 \implies v_1 = 2(1 - v_2)$$

Conservation of kinetic energy:

$$\frac{1}{2} \times 2 = v_2^2 + \frac{1}{2}v_1^2$$

$$v_2^2 + \frac{1}{2}.4.(1 - v_2)^2 = 1$$
$$v_2^2 + 2v_2^2 + 2 - 4v_2 - 1 = 0$$
$$3v_2^2 - 4v_2 + 1 = 0$$
$$(3v_2 - 1)(v_2 - 1) = 0$$

For which one is the trivial situation $v_2 = 1, v_1 = 0$ and the other is the one we are after:

$$v_1 = \frac{4}{3}$$
$$v_2 = \frac{1}{3}$$

3.7 Qu17: Wave Equation and Waves

The wave form is that of a plane wave:

$$y = Asin(kx - \omega t)$$

Now summarising the numerical information we have been given:

$$\lambda = 10$$

$$A = 0.5$$

$$v = 2$$

From the wave equation $v = f\lambda$

$$f = \frac{2}{10} = 0.2Hz$$

The angular velocity of the oscillations ω is given by:

$$\omega = 2\pi f = 0.4\pi$$

$$v = A\omega = 0.4\pi \times 0.5 = 0.2\pi ms^{-1}$$

3.8 Qu18: Force, density and momentum

a) Speed of the water leaving the nozzle:

speed is measured in ms^{-1}.
rate is measured in m^3s^{-1}.
So dimensionally speaking we have:

$$speed = \frac{x}{A}$$

b) i) Water hits the wall and loses all of its horizontal momentum.

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{m\frac{x}{A}}{\Delta t}$$

$$\frac{m}{\Delta t} = x \times \rho$$

$$F = \frac{\rho x^2}{A}$$

ii) This time we have a rebound.

$$F = \frac{m(v - -v)}{\Delta t} = x\rho(2v) = \frac{2\rho x^2}{A}$$

3.9 Qu19: Orbits

a) For a stable orbit we want to equate the gravitational force to the centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$
$$v = \sqrt{\frac{GM}{r}}$$

b) Applying numbers $r_E = 6400km$ and $g = 10ms^{-2}$. First we note that the acceleration due to gravity:

$$F = ma = \frac{GMm}{r^2} \implies a = \frac{GM}{r^2} = g$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{r^2} \times r} = \sqrt{10 \times 6.4 \times 10^6} = 8000ms^{-1}$$

3.10 Qu20: Planck's Energy

a) Starting with the Planck Relation:

$$\Delta E = \frac{hc}{\lambda} = hf \tag{5}$$

Shortest wavelength occurs for the largest energy difference:

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{R(1 - \frac{1}{10^2})} = \frac{100hc}{99R}$$

b) Longest wavelength for smallest energy difference.

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{R(\frac{1}{9^2} - \frac{1}{10^2})} = \frac{8100hc}{19R}$$

c) Level 10 can emit 9 different λs as there are 9 different gaps between the 10 different levels and hence 9 different possibilities for "jumps", level 9 can emit 8 different, etc.

$$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 45$$

3.11 Qu21: Circuits

a) Total resistance between A and B.

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{\frac{3R}{2}}$$

$$R_T = \frac{6R}{13}$$

b) Potential difference V is applied between A and B.

$$V_{BD} = V \times \frac{R}{\frac{3R}{2}} = \frac{2V}{3}$$

Voltage in parallel is same across each branch.

$$P = \frac{V_{BD}^2}{R_{BD}} = \frac{(\frac{2V}{3})^2}{R} = \frac{4V^2}{9R}$$

c) Total resistance between C and D.

We need to re-draw the circuit to make this question simpler. To do this, we can look at each node in the circuit and consider the different paths possible. As there is empty wire between B and C, we can "equate" these two nodes.

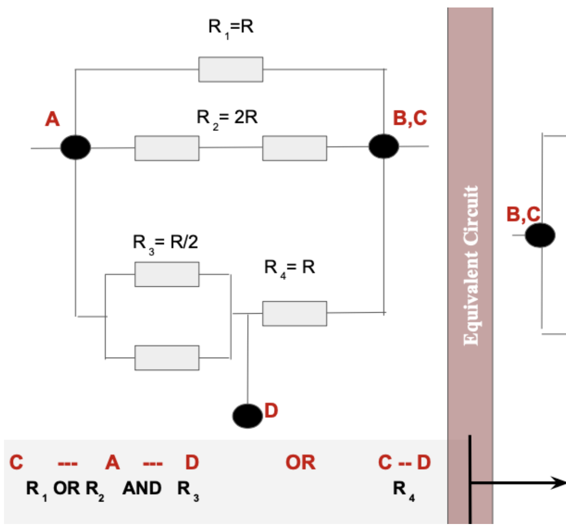

Figure 8: Qu21

We then consider the possible journeys to get from C to D (via nodes) and then identify the resistors we pass through to get to each. Any time we are faced with an "OR" we translate it to drawing a parallel circuit component, and anytime we face an "AND" we have a series connection. This technique can be used to solve the most complex circuits problems but ofcourse there are perfectly valid alternative methods so use what you find easy!

$$\frac{1}{R_T} = \frac{1}{R} + \frac{6}{7R} = \frac{13}{7R}$$

$$R_T = \frac{7R}{13}$$

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