# Physics Aptitude Test (PAT) Unofficial Solutions: 2017

University of Oxford Admissions Test

**Physics, Engineering, Materials Science**

Solutions by PhysOx Initiative
2020

# Contents

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# 1 Foreword

The solutions provided here are by no means an official set of answers; our aim was to provide unofficial solutions in order to provide those studying for the PAT with the means to check their answers as they attempt past paper questions. There is no mark breakdown but we aimed to provide detailed explanations on how to solve the questions and develop a good intuition for them. These solutions have been compiled by our team of Oxford Physics graduates who have all taken the PAT and our team members also have experience in marking the PAT and/or running the PAT summer programmes coordinated by the University of Oxford.

Some general tips for questions:

**1)** Try and **keep your calculations in terms of symbols and letters** until you see your calculations simplifying considerably by substituting numbers. This will genuinely reduce errors in your work and make it way easier for the marker to understand your thought process.

**2)** Give your **variables reasonable names**, for example don't call your initial velocity something like $v_u$ and final velocity something like $v_v$, as you will definitely confuse the life out of everyone looking at your work not to mention yourself.

**3)** Your teachers may say this a lot and many of you probably ignore it but **drawing diagrams really helps!** Sometimes the best way to deal with paragraphs of information is a simple drawing which has all the important bits - it will also save you a lot of time!

**4)** Always **show your working** so it does not look like you picked an answer out of thin air especially for the longer answer questions so it's easier to pick up on exactly where you went wrong if you do go wrong. Fair enough you may have had a moment of next level inspiration but a couple of lines (or words here and there) just outlining your way of thinking really helps. That said, it doesn't have to be an essay!

**5)** Don't feel the need to rush, **relax** yourself and approach the questions. If you get really stuck on one, don't get too put off, just skip it and come back to it later if you get time.

# 2 Section A: Multiple Choice Questions

## 2.1 Answers

| | |
|---|---|
| 1 | D |
| 2 | A |
| 3 | E |
| 4 | D |
| 5 | B |
| 6 | C |
| 7 | C |
| 8 | E |
| 9 | B |
| 10 | A |
| 11 | B |
| 12 | A |

Table 1: Multiple Choice Answers.

## 2.2 Answers Explained

### 2.2.1 Qu1: Differentiation

Product Rule:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \tag{1}$$

for $y = uv$.

### 2.2.2 Qu2: Factorisation

Factorise the quadratic. A useful rule is, that for a quadratic of form

$$Ax^2 + Bx + C = 0$$

to be factorised to

$$(x + a)(x + b) = 0$$

we can say that we want a and b such that

$$a * b = C/A \tag{2}$$

$$a + b = B/A \tag{3}$$

### 2.2.3 Qu3: Sum of Geometric Series

This is a simple problem as long as you are happy with notation. For those not familiar with sum notation take a look at this website, it's nice and clear: http://www.columbia.edu/itc/sipa/math/summation.html
We have a finite geometric series with first term $a = 1$ as $(-e^{-1})^0 = 1$ and also common ratio $r = -e^{-1}$. Sum to n terms is given by:

$$S_n = \frac{a(1 - r^n)}{1 - r} \tag{4}$$

Just substitute your a, r and n=11 (n=10 but we started from n=0 so 10 is the 11th term) and you should get E.

### 2.2.4 Qu4: Logarithms

To lower indices to solve for $x$ we need to use logs.
Get all $a$ terms on one side and $b$ terms on the other.

$$a^{2x+2} = b^{2x}$$

$$(2x + 2)log(a) = 2xlog(b)$$

$$log(a) = (log(b) - log(a))x$$

Then the answer is D.

#### 2.2.5 Qu5: Integrals of Odd and Even Functions

You could integrate these and find out but we know odd functions with symmetric limits will give us 0 (the top half of the graph and bottom half cancel as one is in the positive y axis while the other in the negative) and evens will give us non-zero values as both halves are either positive or negative so no cancellation.

$$Oddfunctions : f(-x) = -f(x) \tag{5}$$

$$Evenfunctions : f(x) = f(-x) \tag{6}$$

Out of the 4 equations all of the limits are symmetric about the origin. This means as I1 and I4 are the only two odd functions, they will be the two zero integrals.

#### 2.2.6 Qu6: Graphing Functions

Key things to notice first:

**1)** There are 3 asymptotes: x= 1,-3 and y=0

so we're looking for the denominator to be 0 at these values of x - so C/A

**2)** The graph goes through (0,-1/3)

still both C and A go through this point

**3)** Let's try a random point
when x= -1, A gives y = +1/2
C however gives y = -1/4

We can see that the graph is negative at this point therefore C is correct.

#### 2.2.7 Qu7: Acceleration

C is the only sensible answer seeing as the astronaut only lightly tosses the ball.

#### 2.2.8 Qu8: EM Spectrum

The full list is: Gamma, X-ray, UV, Visible, Infrared, Micro, Radio

This is probably worth learning along with rough ranges of their wavelengths/ frequencies (if you know one you can find the other using the wave equation $c = f\lambda$.

#### 2.2.9 Qu9: Resistance

Resistances in series add:

$$R_{tot} = R_1 + R_2 + ... \tag{7}$$

Resistances in parallel:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \tag{8}$$

So the first parallel bit collapses to a resistor with resistance

$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{2R}$$

$$R_1 = \frac{2R}{3}$$

Then the circuit remaining is a series circuit with resistances $R_1$ and R:

$$R_{tot} = \frac{2R}{3} + R = \frac{5R}{3}$$

From $V = IR$

$$I = \frac{3V}{5R}$$

This is the current.

#### 2.2.10 Qu10: Capacitance

The key equation to remember here for capacitance is:

$$C = \frac{\epsilon_0 A}{d} \tag{9}$$

for A the plate area, d the distance between parallel plates.
Therefore from proportionality if we half the area we half the capacitance.

#### 2.2.11 Qu11: Pulleys

We need the forces up to balance the forces down (around the mass). So we have the two segments of rope around the first pulley acting like two equal forces going up and mg going down. We assume tension in string is the same (the rope isn't stretchy) so the force F applied gets double input in the balancing act (due to the two arms coming out of pulley 1). Therefore $mg = 2F$ so B.

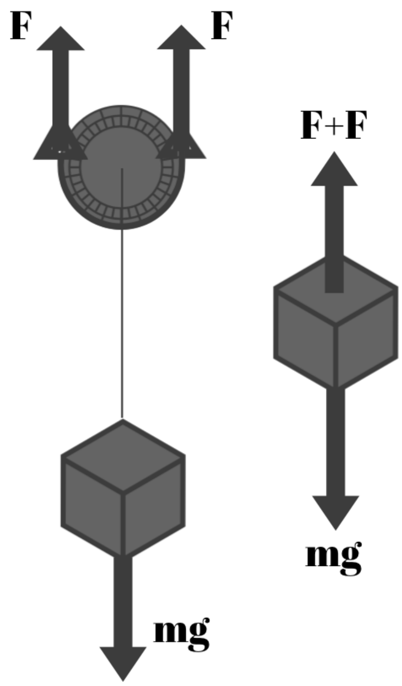



Figure 1: Qu11

### 2.2.12 Qu12: Charged Particle Motions

We are told the particle comes to a stop after time t so final momentum (and velocity is 0). This means from Newton II:

$$F = \frac{\Delta p}{\Delta t} = \frac{p_i}{t}$$

Force due to a charged particles:

$$F = qE = \frac{qV}{d}$$

Combining all of that we get A:

$$p = \frac{qVt}{d}$$

# 3 Section B: Long Answer Questions

### 3.0.1 Qu13: Binomial Expansion

$$\binom{n}{k} = {}^{n}C_k = \frac{n!}{k!(n-k)!} \tag{10}$$

This is the combinations formula! Also worth remembering and making note of the permutations formula:

$${}^{n}P_k = \frac{n!}{(n-k)!} \tag{11}$$

$$\begin{aligned}(3+2x)^5 &= \\ 3^5 + {}^5C_1(3)^4(2x)^1 + {}^5C_2(3)^3(2x)^2 + {}^5C_3(3)^2(2x)^3 + {}^5C_4(3)^1(2x)^4 + (2x)^5 &= \\ 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5\end{aligned}$$

### 3.0.2 Qu14: Probability

For this question, make sure you're happy with the way the AND rule works with probabilities.

Always highlight the relevant equation to organise your thoughts:

Person A - 0.5
Person B - 0.75
Person C - 0.2

**a)** All three busy: $0.5 * 0.75 * 0.2 = 0.075$

**b)** All three not busy: $(1-0.5)*(1-0.75)*(1-0.2) = 0.1$

### 3.0.3 Qu15: Forces and Motion

Spring Constant k, Natural length L, masses m and M, friction coeff $\mu_s$
To move m we need the force due to the tension in the spring to be greater than friction (From Newton I we need imbalance of forces). This critical force (when resistance = to the pulling force) corresponds to a critical extension which we can find by applying Hooke's Law.

Note that the force of friction can be found from the coefficient as $F_{frm} = \mu_s mg$ and for the mass M, $F_{frM} = \mu_s Mg$.

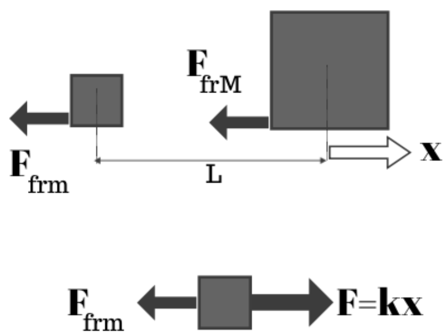



Figure 2: Qu15

Now Hooke's Law gives us:

$$F = kx \tag{12}$$

The displacement in M is the extension in the spring and hence is the pulling force on m. The friction on M doesn't play a role in the motion of the mass m as it doesn't affect the tension. We want the tension = friction due to the small mass for the critical condition for movement.

$$kx = \mu_s mg$$

$$x = \frac{\mu_s mg}{k}$$

### 3.0.4 Qu16: Cones and Spheres

Define radius of cone to be $r_{cone}$

$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$V_{cone} = \frac{1}{3}\pi r_{cone}^2 h = \frac{2r}{3}\pi r_{cone}^2$$

as h = 2r.
Now set $V_{cone} = V_{sphere}$ and rearrange for $r_{cone}$.

$$r_{cone} = \sqrt{2}r$$

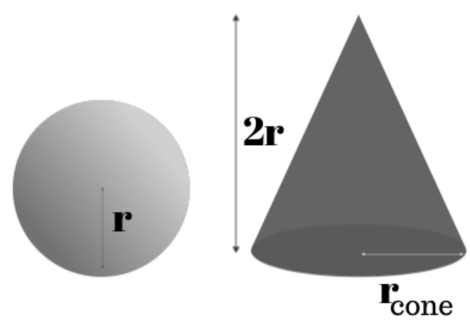



Figure 3: Qu16

### 3.0.5 Qu17: Forces and Motion

**a)** The first part is a dimensional analysis question - just look at the units of everything else and equate.

$$[\alpha v^2] = kgms^{-2}$$

We know the units of $v$ is $ms^{-1}$ substituting and equating as above.

$$[\alpha] = \frac{kgms^{-2}}{m^2s^{-2}} = kgm^{-1}$$

**b)** Terminal velocity occurs when weight = air resistance/drag force up so the object falling reaches equilibrium and the velocity stops increasing (constant). Identifying the weight term as $mg$ and the resistance term as $\alpha v^2$ in the equation, we equate them and rearrange to find $v_{term}$.

$$v_{term} = \sqrt{\frac{mg}{\alpha}}$$

.

**c)** Key assumptions: terminal velocity isn't reached before the ground. It is reached just about as she hits the ground.

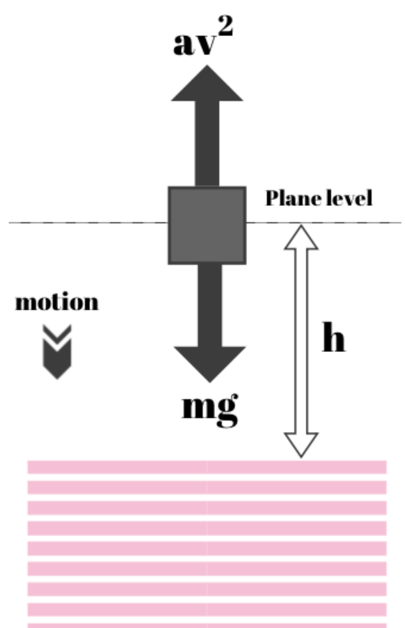



Figure 4: Qu17

Energies involved are GPE and KE.

$$\Delta GPE = mgh$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2}{\alpha}$$

Where we applied b) above.

Air resistance does work to reduce the loss of GPE and reduce the velocity (or KE) of the object so:

$$Workdone = mgh - \frac{m^2 g}{2\alpha}$$

### 3.0.6 Qu18: Waves and Interference

**a)** Angular frequency:

$$\omega = 2\pi f$$

Wave equation:

$$v = f\lambda$$

Therefore:

$$v = \frac{\omega\lambda}{2\pi}$$

**b)** Consider $y_1 + y_2$

$$y_{tot} = Acos(\frac{2\pi x}{\lambda_1} - \omega_1 t) + Acos(\frac{2\pi x}{\lambda_2} - \omega_2 t) \tag{13}$$

$$= 2Acos[\frac{(\lambda_2+\lambda_1)\pi x}{\lambda_1\lambda_2} - \frac{(\omega_1+\omega_2)t}{2}]cos[\frac{(\lambda_2-\lambda_1)\pi x}{\lambda_1\lambda_2} - \frac{(\omega_1-\omega_2)t}{2}] \tag{14}$$

This was simplified using the identity given. This equation has two parts. The wavenumber $k = 2\pi/\lambda$ is the coefficient of x in each part of the wave's equation. From the superposed waves, we have a long wavelength term and a short wavelength term. The long wavelength term is an "envelope" wave (wavelength $L_2$) which envelopes shorter oscillations (wavelength $L_1$).

$$L_1 = 2\frac{\lambda_2\lambda_1}{\lambda_2+\lambda_1}$$

$$L_2 = 2\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}$$

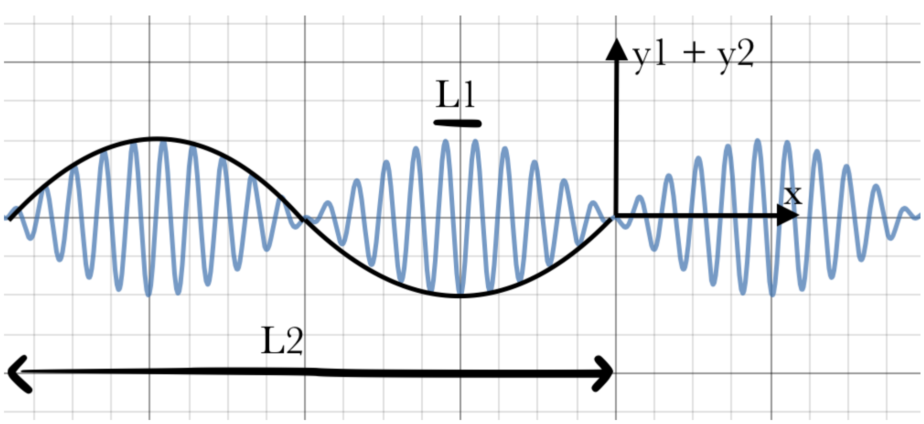



Figure 5: Qu18

**c)** Sound disappears at nodes in the diagram in b). This is half of the long wavelength:

$$L_2/2 = \frac{\lambda_2\lambda_1}{\lambda_2 - \lambda_1}$$

The frequency is that corresponding to the short wavelength oscillations:

$$f = \frac{v}{L_1} = \frac{\omega_1(\lambda_2 + \lambda_1)}{4\pi\lambda_2}$$

### 3.0.7 Qu19: Parametric Equations

There is more than one way of doing this. For parametric equations you can try and first get rid of the t-dependent terms by using the identity $cos^2x + sin^2x = 1$ and then some rearranging before setting y to 0 and solving for x. However, with this question it may be quicker to look at the y equation and set it to 0 to find the condition for the t-dep. term and then apply this to the x equation:

$$a(\sqrt{3} - 2cos(\omega t)) = 0$$
$$cos(\omega t) = \frac{\sqrt{3}}{2}$$

for which solutions are:

$$\omega t = \frac{\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$$

so that

$$sin(\omega t) = 1/2, -1/2$$

Substituting into the x parametric equation for the two cases:

$$x = a(\frac{\pi}{6} + 2\pi n - \frac{1}{2})$$

$$x = a(\frac{\pi}{6} + 2\pi n + \frac{1}{2})$$

Where n is 0, 1 , 2 , 3, etc.

### 3.0.8 Qu20: Newton's Law of Gravitation and Orbits

For the two star system, the stars are opposite each other in the circle (as is our assumption) therefore from Newton's Law of Gravitation and Centripetal Motion (equating the two):

$$\frac{mv_2^2}{R} = \frac{Gmm}{(2R)^2}$$

$$v_2^2 = \frac{Gm}{4R}$$

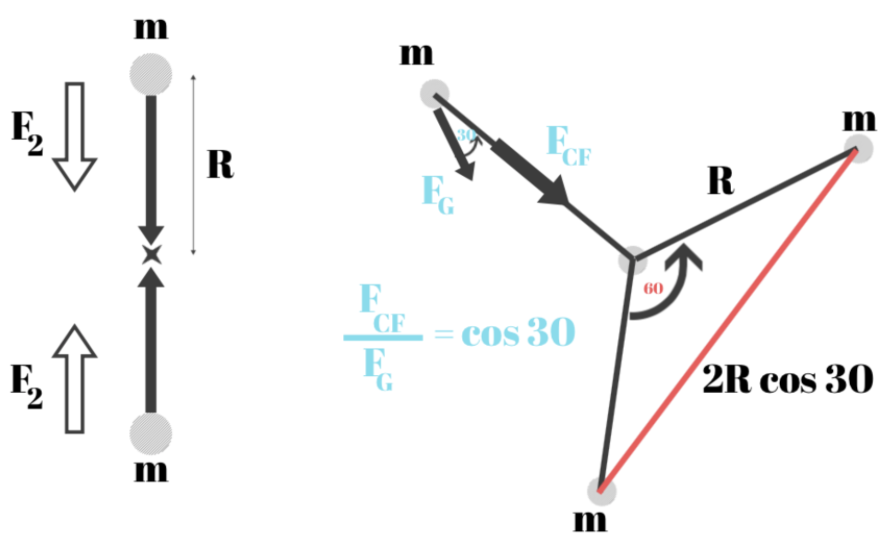



Figure 6: Qu20

Similarly for the three-star system this time the separation isn't 2R but is $2Rcos(30)$ from geometry. We also have contributions from 2 stars and as centripetal acceleration is due to the force perpendicular to the velocity of the object the contribution of the gravitational force that we want to equate to is $F_{cf} = F_g cos(30)$:

$$\frac{mv_3^2}{R} = \frac{Gmm}{(2Rcos(30))^2}.2.cos(30)$$

$$v_3^2 = \frac{\sqrt{3}Gm}{3R}$$

Taking a ratio of the two speeds and rearranging for $v_3$:

$$v_3 = \frac{2}{3^{0.25}} v_2$$

### 3.0.9 Qu21: Integration and Differentiation

Consider the integration first. In the integral, we see that we are integrating with respect to dx so t behaves like a constant.

$$\frac{d}{dt}t^4\int_0^{2t^2} x^4 dx$$

$$\frac{d}{dt}t^4[\frac{x^5}{5}]_0^{2t^2}$$

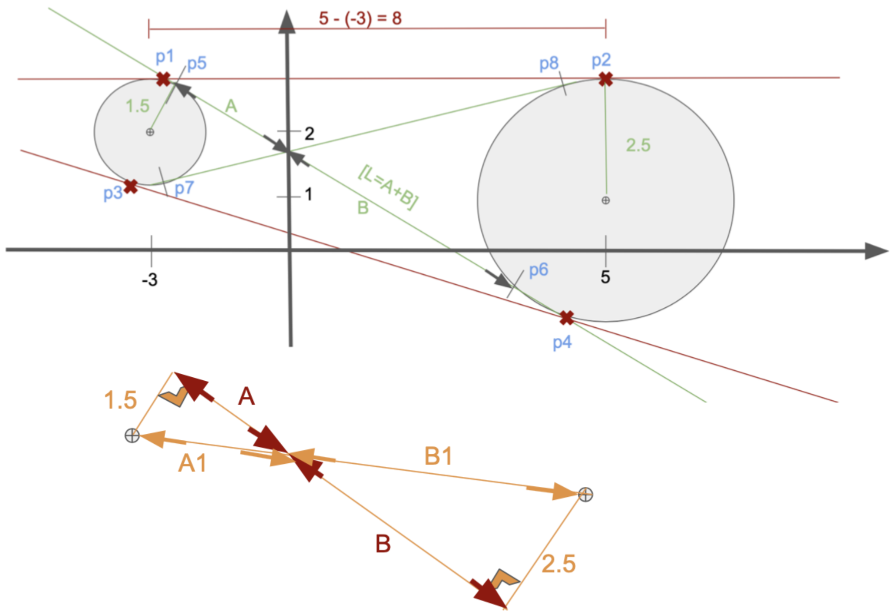
Apply the substitution in the square brackets

$$\frac{d}{dt}(\frac{32}{5}t^{14})$$

Finally differentiate what's inside wrt. t.

$$= \frac{448}{5}t^{13}$$

### 3.0.10 Qu22: Circles and Tangents




$$A_1 + B_1 = ((5+3)^2 + (2-1)^2))^{1/2} = \sqrt{65}$$

Figure 7: Qu22

Simplifying the two equations to a more familiar form by completing the square for both x and y variables:

$$(x+3)^2 + (y-2)^2 = (\frac{3}{2})^2$$

$$(x-5)^2 + (y-1)^2 = (\frac{5}{2})^2$$

From a circle theorem if two tangents are drawn to a circle, then they have equal tangent segments.
This means that $p_1 - p_2$ and $p_3 - p_4$ are the same length and equally $p_5 - p_6$ and $p_7 - p_8$ are the same length.
a) From the diagram we find that $p_1 - p_2$ is a straight line with length 8 (difference between the centres of the circles). Therefore $p_3 - p_4$ is also length 8.

b) From the diagram we seen the similar triangles 1 and 2.
This means that $\frac{1.5}{A} = \frac{2.5}{B}$ or $B = \frac{5}{3}A$ and the length L = A + B

$$L = \frac{8}{3}A$$

We also know that the distance between the centres are $\sqrt{8^2+1^2} = \sqrt{65}$.
From the relation before (similar triangles), $A_1 = \frac{3}{8}\sqrt{65}$.
From Pythagoras, $A^2 = A_1^2 - 1.5^2$ therefore applying to the equation for L before.

$$L = \frac{8}{3}\sqrt{A_1^2 - 1.5^2} = 7$$

From the circle theorem this is the length between $p_5 - p_6$ and $p_7 - p_8$. Note: There are different ways of doing this so chances are if you ended up at the same answer your method is completely fine even if it looks completely different!

### 3.0.11 Qu23: Refraction

This is going to be lots of Snell's Law and triangles.
The ring will appear to stop descending past the critical angle in the right tank.
Condition for that is:

$$n_2 = n_1 sin(\frac{\pi}{2} - \theta_1) = n_1 cos(\theta_1)$$

We need all of this in terms of only depths and refractive indices (no angles).
Let's consider some other relations:
For the air-tank1 boundary:

$$sin(\theta_0) = n_1 sin(\theta_1)$$

From the identity $sin^2(\theta) + cos^2(\theta) = 1$ we can rearrange the above two equations and obtain:

$$sin^2(\theta_0) = n_1^2 - n_2^2$$

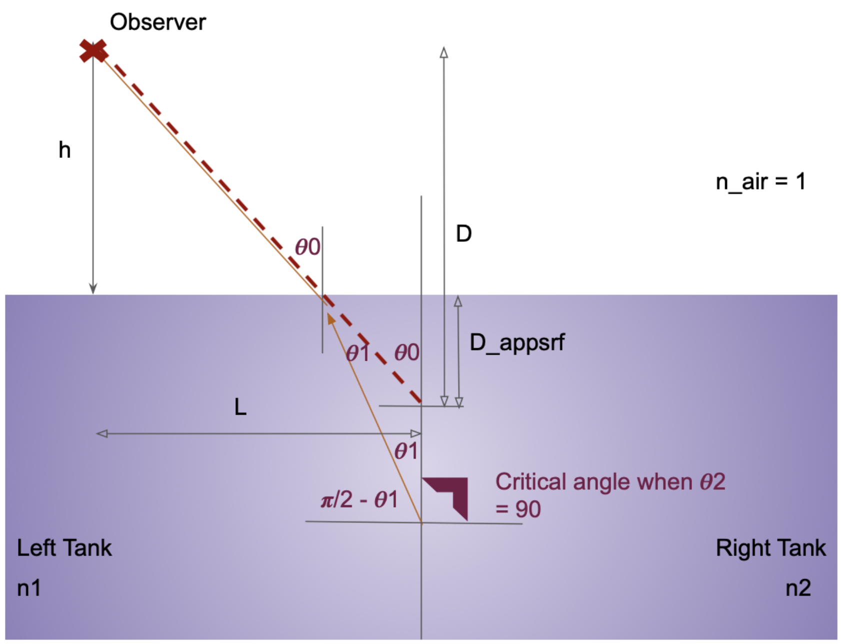



Figure 8: Qu23

If we define the apparent depth from the observer as D then from Pythagoras we also have:

$$sin(\theta_0) = \frac{L}{\sqrt{L^2 + D^2}}$$

In terms of refractive indices, combining the two equations above and rearranging for D:

$$D^2 = \frac{L^2(1 + n_1^2 - n_2^2)}{n_1^2 - n_2^2}$$

Therefore apparent depth from surface = D - h

$$D_{appsrf} = L\sqrt{\frac{1 + n_1^2 - n_2^2}{n_1^2 - n_2^2}} - h$$

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