Physics Aptitude Test (PAT) Unofficial Solutions: 2017 (Specimen)

University of Oxford Admissions Test

Physics, Engineering, Materials Science

Solutions by PhysOx Initiative

2020

Contents

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1 Foreword

The solutions provided here are by no means an official set of answers; our aim was to provide unofficial solutions in order to provide those studying for the PAT with the means to check their answers as they attempt past paper questions. There is no mark breakdown but we aimed to provide detailed explanations on how to solve the questions and develop a good intuition for them. These solutions have been compiled by our team of Oxford Physics graduates who have all taken the PAT and our team members also have experience in marking the PAT and/or running the PAT summer programmes coordinated by the University of Oxford.

Some general tips for questions:

1) Try and **keep your calculations in terms of symbols and letters** until you see your calculations simplifying considerably by substituting numbers. This will genuinely reduce errors in your work and make it way easier for the marker to understand your thought process.

2) Give your **variables reasonable names**, for example don't call your initial velocity something like v_u and final velocity something like v_v, as you will definitely confuse the life out of everyone looking at your work not to mention yourself.

3) Your teachers may say this a lot and many of you probably ignore it but **drawing diagrams really helps!** Sometimes the best way to deal with paragraphs of information is a simple drawing which has all the important bits - it will also save you a lot of time!

4) Always **show your working** so it does not look like you picked an answer out of thin air especially for the longer answer questions so it's easier to pick up on exactly where you went wrong if you do go wrong. Fair enough you may have had a moment of next level inspiration but a couple of lines (or words here and there) just outlining your way of thinking really helps. That said, it doesn't have to be an essay!

5) Don't feel the need to rush, **relax** yourself and approach the questions. If you get really stuck on one, don't get too put off, just skip it and come back to it later if you get time.

2 Section A: Multiple Choice Questions

2.1 Answers

1	A
2	B
3	D
4	A
5	C
6	C
7	D
8	C
9	A
10	B
11	D
12	D

Table 1: Multiple Choice Answers.

2.2 Answers Explained

2.2.1 Qu1: Binomial Expansion

$$\binom{n}{k} = {}^nC_k = \frac{n!}{k!(n-k)!} \tag{1}$$

This gives us the coefficients of each of the terms in the expansion for a power n and the k^{th} power term in the expansion.

$$(1+4x+6x^2+4x^3+x^4)(1-6x+15x^2-20x^3+15x^4-6x^5+x^6)$$

We can then consider the terms which would multiply to give us x^5.

$$(1\times -6x^{-3}) + (4x\times 15x^4) + 6x^2(-20x^3) + 4x^3(15x^2) + x^4(-6x)$$
$$= -6+60-120+60-6$$
$$= -12$$

Therefore the answer is A.

2.2.2 Qu2: Logarithms

Generally:

$$log_a b = c \implies a^c = b \tag{2}$$

Then we can simplify and solve the equation:

$$log_2 x + 2 = 2$$
$$2^{log_2 x} = 2^0$$
$$x = 1$$

The answer is B.

2.2.3 Qu3: Differentiation

This is a combination of the Product and Chain Rules of differentiation.

$$y = xsin(x^2)$$
$$\frac{dy}{dx} = sin(x^2) + x.2x.cos(x^2)$$
$$\frac{dy}{dx} = sin(x^2) + 2x^2cos(x^2)$$

This is D!

2.2.4 Qu4: Gradients

This one is a super easy one - just your standard gradient of a straight line.

$$m = \frac{\Delta y}{\Delta x} = \frac{8 - (-2)}{-5 + 4} = \frac{10}{-1} = -10$$

The answer is A.

2.2.5 Qu5: Arithmetic Progression

First term $a = 1$.
Common difference $d = 1$.
Number of terms $n = 100$.
The sum to n terms for an arithmetic progression is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d) \tag{3}$$

$$S_n = 50(2 + 99) = 50(101) = 5050$$

You don't need to know this equation to solve this equation because you can actually spot that if you sum each number from the start with a number at the end and get closer to the middle the sum of each pair of numbers is 101 and this carries on till you reach $50, 51$. The formula is just handy for if you feel lazy and don't want to risk spotting patterns!

The answer is C.

2.2.6 Qu6: Forming Numbers

We have access to 5 numbers and we are told we need to make a number greater than 5000. There are two possibilities to consider:

a) We have *5 numbers* to choose from.

In this case we are allowed any choice of numbers for the 5 digits as long as we remember no repetitions:

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$$

So in this scenario we get 120 possible numbers greater than 5000.

b) We have *4 numbers* to choose from.

Here we need to be more careful. The first digit can only be $5, 6 or 7$ but after than we can choose any of the 4 remaining digits to fill up the 3 remaining digits as long as no repeats.

$$3 \times 4 \times 3 \times 2 = 72$$

So for 4 digits we have 72 combinations.

So the overall is now $72 + 120 = 192$.
The answer is C.

2.2.7 Qu7: Orbits and the Solar System

Take each statement in turn:

1. Duration of day is related to rotation around a planet's own axis so this is wrong.

2. From Kepler's $3rd$ Law, the duration of the year should increase: square of orbital period $\propto$ cube of semi-major axis of orbit. So this is **correct**.

3. The size/ volume of the planets increases - well this one is clearly not true!

4. Number of moons don't really increase.

5. Planets do seem to change from rocky to gas giants so **correct**.

So 2 and 5 are correct and the answer is D.

2.2.8 Qu8: EM Spectrum

Usually it is easier to remember the wavelengths (or at least the ballparks). Assuming we know some rough estimates of wavelengths, we can use the wave equation to convert the frequency to a wavelength.

$$v = f\lambda \tag{4}$$

EM waves travel at the speed of light c.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^9} = 3 \times 10^{-3}$$

This is of the order of millimeters so most likely in the microwave region of the spectrum: C.

2.2.9 Qu9: Orbits

The object has the same velocity in the orbit as the ISS, no thrust and no loss in energy (assuming no frictional forces) so A.

2.2.10 Qu10: Orbits and Kepler's Laws

Let's use Kepler's Third Law and proportionalities:

$$T^2 \propto r^3 \tag{5}$$

As a result we can say the ratio of these terms are a constant:

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$$

We also have $R_2 = R_1/2$.

$$T_2^2 = T_1^2(\frac{R_2}{R_1})^3$$

$$T_2^2 = \frac{24^2}{8} \implies T_2 \approx 8.5$$

The answer is B.

2.2.11 Qu11: Radioactivity

If A has a half-life of τ then B has a half-life of 2τ. If B has an initial population of X_0 then A has an initial population of $2X_0$.

$$X_A = 2X_0 2^{-\frac{t}{\tau}}$$

$$X_B = X_0 2^{-\frac{t}{2\tau}}$$

$$\frac{X_A}{X_B} = 2 \times 2^{-\frac{t}{2\tau}} = \frac{1}{2}$$

$$2^{-t/2\tau} = 2^{-2}$$

$$t/2\tau = 2$$

$$t = 4\tau = 12$$

12 days so D.

2.2.12 Qu12: Cubes

All cubes that are 1 cube inside each face will not be painted. So if we have a $5 \times 5 \times 5$ cube then the $3 \times 3 \times 3$ cube inside will not be painted: 27 cubes will be completely unpainted - D.

3 Section B: Long Answer Questions

3.0.1 Qu13: Probabilities

Let's write down what we know:

$$2g = y$$
$$2y = r$$
$$2r = b$$

As ratios:

$$y : g : r : b$$
$$2 : 1 : 4 : 8$$

Now we just read it off (fractions are out of 15):

a) $\frac{8}{15}$
b) $\frac{4}{15}$
c) $\frac{2}{15}$
d) $\frac{1}{15}$

3.0.2 Qu14: Projectiles

Projectile velocity $200ms^{-1}$ at some angle θ and rail car at speed $100ms^{-1}$.

$$200cos\theta = 100$$
$$cos\theta = \frac{1}{2}$$
$$\theta = 60^\circ$$

3.0.3 Qu15: Geometric Series

We have a geometric series with first term $a = 1$ and common ration $r = e^{-x}$. The sum to infinity:

$$S_\infty = \frac{a}{1-r} \tag{6}$$
$$S_\infty = \frac{1}{1-e^{-x}}$$

Valid for $|r| < 1$:

$$|e^{-x}| < 1$$
$$e^{-2x} < 1$$
$$-2x < 0 \implies x > 0$$

3.0.4 Qu16: Acceleration of Charges

This is essentially just a SUVAT question.

The screen is a distance $D = 0.4m$ away. We also know that there is no external force in the horizontal direction so for an electron released from the tube with velocity v_h, the electron retains this horizontal speed all the way to the screen. We can then find the time to get to the screen:

$$t = \frac{D}{v_h}$$

Now to can look at what we know about the vertical direction:

$$s = ?$$

$$u = 0$$

$$v =$$

$$a = -10$$

$$t = \frac{D}{v_h}$$

We assume that the initial vertical velocity is zero. We can then use $s = ut + \frac{1}{2}at^2$ with $u = 0$.

$$s = \frac{1}{2}(-10)(\frac{D}{v})^2 \tag{7}$$

We can find the velocity v_h from energy considerations: the kinetic energy of an electron is equal to the energy due to acceleration by the applied electric field of the electron gun.

$$\frac{1}{2}mv_h^2 = eV$$

$$v_h = [\frac{2eV}{m}]^{\frac{1}{2}} = 4 \times 10^6 ms^{-1}$$

Applying numbers and substituting back into the expression for s:

$$s = 5 \times 10^{-14} m$$

3.0.5 Qu17: Integration

(a) Integration by Substitution

$$\int_0^{\frac{\pi}{2}} \frac{cosx}{1 + sinx} dx$$

Substitute $u = 1 + sinx$ and $\frac{du}{dx} = cosx$. The new limits are then: $u_2 = 2$ and $u_1 = 1$.

$$\int_1^2 u^{-1} du = [lnu]_1^2 = ln2$$

(b) We start by factorising the denominator first and then doing our normal partial fractions procedure to simplify the expression we need to integrate.

$$x^2 + 6x + 8 = (x+4)(x+2)$$

$$\frac{x}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$$
$$= \frac{A(x+2) + B(x+4)}{(x+4)(x+2)}$$

Equating coefficients:

$$A + B = 1$$
$$2A + 4B = 0 \implies A = -2B$$
$$B = -1, A = 2$$

Now going back to the integral:

$$\int_0^2 \frac{2}{x+4} - \frac{1}{x+2} dx$$

$$[2ln(x+4) - ln(x+2)]_0^2 = 2ln6 - ln4 - 2ln4 + ln2$$
$$= ln(36/4) - ln(16/2)$$
$$= ln(9/8)$$

3.0.6 Qu18: Springs

(a) Normally the period is given by:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{8}$$

where $k = springconstant$ and $T = period$.

In Series: $F = k_{eff}x$ (normally from Hooke's Law)

Overall two-spring system is displaced by x so each spring of spring constant k is displaced by $\frac{x}{2}$ or we can say that the overall system has an effective spring constant of $k_{eff} = \frac{k}{2}$.

$$F = \frac{k}{2}x$$

$$T_{series} = 2\pi\sqrt{(\frac{m}{k})} = \sqrt{2}T$$

In Parallel: Again considering $F = k_{eff}x$ This time each spring extends by x so

$$F = k(x + x) = 2kx$$

Therefore

$$T_{parallel} = 2\pi\sqrt{(\frac{m}{2k})} = \frac{T}{\sqrt{2}}$$

(b) If the surface gravity is changed then the period will still remain unchanged as there is no dependence on g.

3.0.7 Qu19: Series and Parallel Circuits

This one is a pretty standard question but potentially quite prone to errors so take it step by step!

The two resistors in parallel can be replaced by $R/2$ so the top branch of the circuit has a resistance of:

$$R + \frac{R}{2} = \frac{3R}{2}$$

The three-branch parallel circuit in the bottom branch has a net resistance:

$$\frac{1}{R_3} = \frac{1}{2R} + \frac{1}{R} + \frac{1}{R}$$

$$= \frac{1 + 2 + 2}{2R} = \frac{5}{2R}$$

Therefore overall resistance in lower branch is $\frac{2R}{5} + R = \frac{7R}{5}$. And the overall resistance between A and B:

$$\frac{1}{R_T} = \frac{2}{3R} + \frac{5}{7R} = \frac{14 + 15}{21R} = \frac{29}{21R}$$

$$R_T = \frac{21}{29}R$$

3.0.8 Qu20: Geometry and Areas

$$Area = A_{\Delta} - 3A_{sector}$$

$$3A_{sector} = (\frac{\pi/3}{2\pi}).3\pi r^2 = \frac{\pi r^2}{2}$$

We can find the area of the triangle:

$$A = 0.5 \times b \times h \times 2$$

$$A = (2rsin(60))\frac{2r}{2} = \sqrt{3}r^2$$

$$Area = r^2(\sqrt{3} - \pi/2)$$

3.0.9 Qu21: Diodes

Diodes are devices which have infinite resistance in one direction and so makes sure current flows in a specific direction in a circuit.

In this circuit, we have a network of two diodes so the IV graph would look like the following. The $0.7V$ is around the typical threshold voltage for silicon diodes (a popular choice). Note other semiconductor materials can also be used and they will have a different corresponding threshold voltages. This $\sim 0.7V$ threshold is a value that is worth just learning - after all it's just one number!

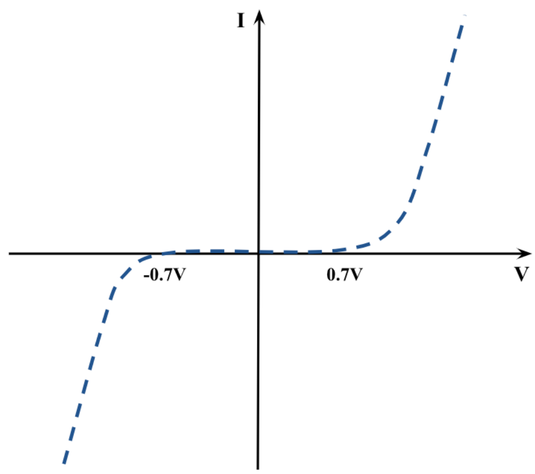

Figure 1: Qu21

For small signals the current should go straight through to the amplifier.

For high voltage, a discharge current will flow through the diode system rather than reaching EF so the instrument is protected.

3.0.10 Qu22: Volumes of Spheres and Cylinders

Total volume needed:

$$V_{snowman} = \frac{4}{3}\pi(r^3 + (2r)^3)$$

$$= \frac{4}{3}\pi(9r^3) = 12\pi r^3$$

Therefore:

$$V_{clay} = \pi(\frac{r}{2})^2 l = \frac{\pi r^2}{4} l$$
$$\frac{l}{4} = 12r$$
$$l = 48r$$

3.0.11 Qu23: Electric Motor

A good habit to get into when dealing with long worded questions is to just pick out the information that is important:

$$Voltage V = 230V$$
$$Diameter D = 5cm$$
$$mass m = 100kg$$
$$speed u = 0.5ms^{-1}$$

(a) Electric current driving the motor:

$$P = \frac{mgh}{t} = \frac{100 \times 10 \times 0.5}{1} = 500W$$
$$I = \frac{P}{V} = \frac{500}{230} = \frac{50}{23} A$$

(b) $v = r\omega$ and we have $r = D/2$.
To lift by $0.5ms^-1$ each string goes up by the same amount so the winding reel must pull at a rate $0.5 \times 3 = 1.5ms^{-1}$. Therefore:

$$\omega = \frac{1.5}{D/2} = \frac{15}{0.25} = 60rads^{-1}.$$

(c) Force

$$F = \frac{P}{v} = \frac{500}{1.5} = 333N$$

3.0.12 Qu24: Inequalities

We will start with a quick sketch and define two areas A and B so that the area of the region defined is $Area = A + B$.

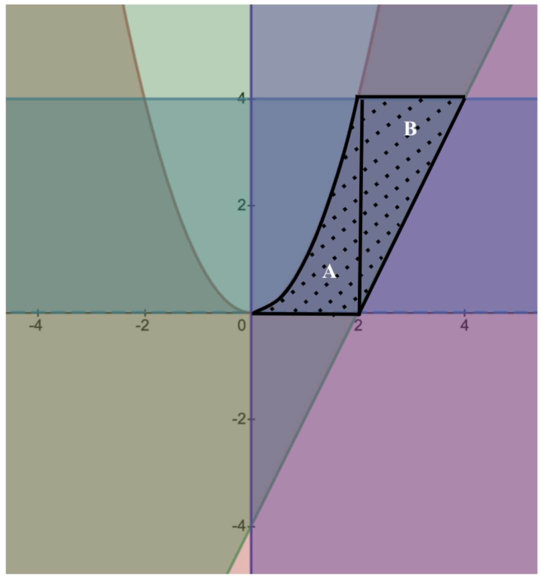

Figure 2: Qu24

We can calculate the areas.

$$A = \int_0^2 x^2 dx = [\frac{x^3}{3}]_0^2 = \frac{8}{3}$$

$$B = \frac{1}{2} \times 2 \times 4 = 4$$

So the total area:

$$Area = \frac{8}{3} + 4 = \frac{20}{3}$$

3.0.13 Qu25: Pulleys

(a) Acceleration of the masses and the tension in the string.

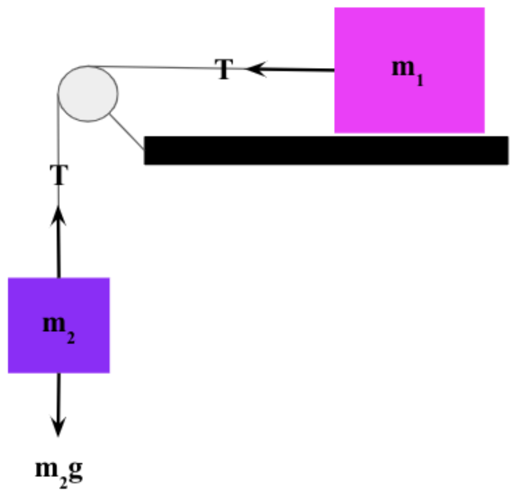

Figure 3: Qu25a

Consider Newton's Second Law for constant mass ($F = ma$) applied to each mass.

$$1.T = m_1 a$$

$$2.m_2 g - T = m_2 a$$

Combine the two and solve simultaneously:

$$m_2 g - m_1 a = m_2 a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2}$$

(b) We now have to consider an additional (frictional) force.

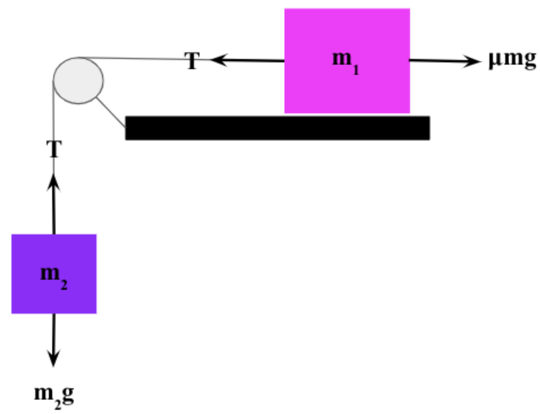

Figure 4: Qu25b

Again same procedure, let's look at the equations of motion for each mass.

$$1. T = \mu mg = m_1 a$$

$$2. m_2 g - T = m_2 a$$

Combine and solve simultaneously:

$$m_2 g - \mu m_1 g = (m_1 + m_2)a$$

$$a = \frac{g(m_2 - \mu m_1)}{(m_1 + m_2)}$$

Stick back into one of the equations to find the tension:

$$T = m_2 g - m_2 a$$

$$T = m_2 g(1 + \frac{\mu m_1 - m_2}{m_1 + m_2})$$

For acceleration we need $T > \mu m_1 g$ and $T < m_2 g$

$$\mu m_1 g < T < m_2 g$$

so

$$\mu m_1 < m_2$$

3.0.14 Qu26: Tangents to Circles

First thing to do here is make a sketch and label what you know.

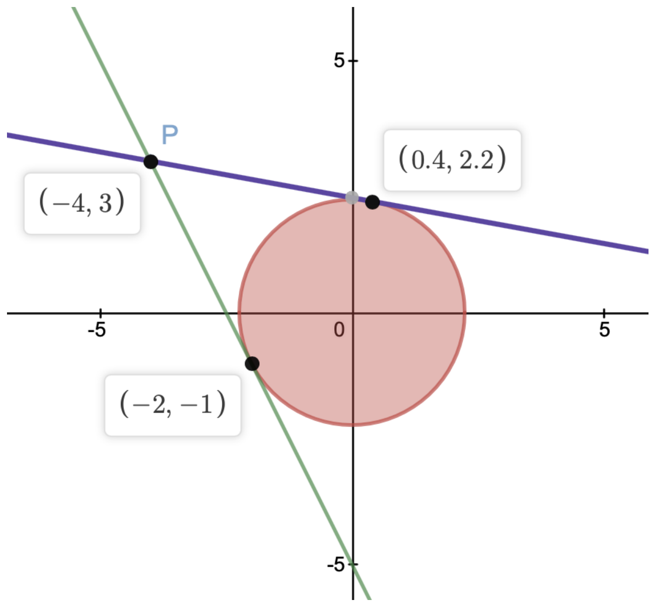

Figure 5: Qu26

Now we work through the geometry step by step. Let's label the point at which the tangent hits the circle in the positive quadrant with coordinates (a, b). Then because this point lies on the circle:

$$a^2 + b^2 = 5$$

Applying Pythagoras between this point (the point where the tangent hits the circle forms a right angle to the circle from its centre), the point $(-4, 3)$ and the origin, we get:

$$(a+4)^2 + (b-3)^2 = 20$$
$$a^2 + b^2 + 8a - 6b = -5$$

Combining with the first circle expression we wrote down.

$$8a - 6b = -10$$

We can reapply this to the first circle expression and remove our b dependence:

$$a^2 + (\frac{4a+5}{3})^2 + 8a - 6(\frac{4a-5}{3}) = -5$$

$$9a^2 + 16a^2 + 40a + 25 + 72a - 72a - 90 = -5$$
$$5a^2 + 8a + 4 = 0$$
$$(5a-2)(a+2) = 0$$

$$a = \frac{2}{5}, a = -2$$
$$b = \frac{11}{5}, b = -1$$

Now we can find the gradients which is just:

$$m_1 = \frac{\frac{11}{3} - 3}{\frac{2}{5} + 4} = \frac{11 - 15}{2 + 20} = \frac{-2}{11}$$
$$y - 3 = \frac{-2}{11}(x + 4)$$

You could leave it like that but we can make it look slightly prettier:

$$2x + 11y - 25 = 0 \tag{9}$$

And the second expression:

$$m_2 = \frac{-1 - 3}{-2 + 4} = 2$$
$$y - 3 = -2(x + 4)$$
$$2x + y + 5 = 0 \tag{10}$$

3.0.15 Qu27: Optical Fibre

This question is quite popular and is probably a good one to make sure you understand and know very well! Also use diagrams where you can - they are your friends and put a smile on examiners' face!

(a) Total internal reflection (TIR) occurs when all light is reflected at a boundary (between two materials with different refractive indices). This occurs when light enters at an angle (to the normal) greater than a critical angle (the "threshold angle" after which TIR will occur defined by when incident light bends an angle of 90° from the normal). The diagram below shows this.

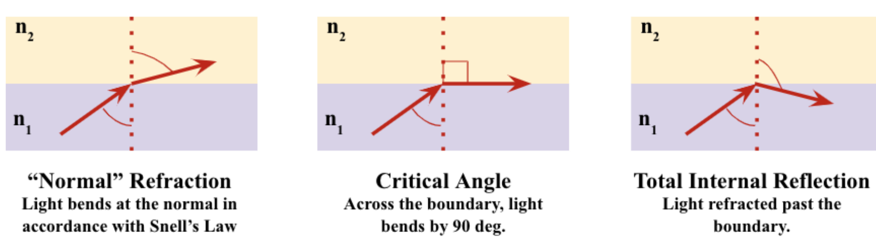

Figure 6: Qu27a

(b) Snell's Law:

$$\begin{aligned} n_1 sin\theta_1 &= n_2 sin\theta_2 \\ n_1 sin\theta_c &= n_2 sin90 \\ sin\theta_c &= \frac{n_2}{n_1} \end{aligned} \quad (11)$$

(c) Optical Fibres

Let's apply Snell's Law from (b) to this particular case. We have two boundaries to consider: the core/cladding boundary and the outside/core boundary. The critical condition comes from the core/cladding boundary, then we have to use geometry to find the resulting limitation on θ_{max}. Remember, we want our expression to be in terms of only variables we, as humans, have knowledge of or can control and change ourselves. In this case we have knowledge of our materials so we know $n_{core}, n_{clad}, n_{air}$ and we can control θ_{max}. Any other angle we need to remove from our final equation because we have no control/independent understanding of them. Always start with a diagram:

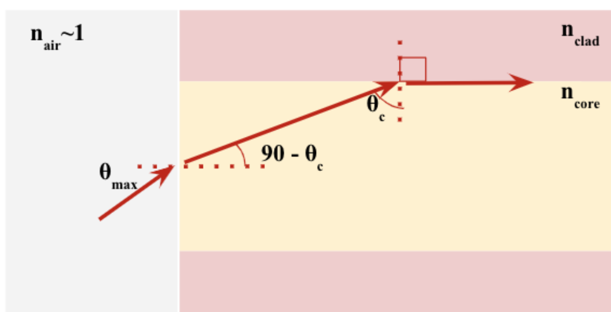

Figure 7: Qu27c

Our critical condition must be applied to the core-cladding boundary where we need to have total internal reflection (TIR) so we don't get losses through the wire due to refraction.

$$sin\theta_c = \frac{n_{clad}}{n_{core}}$$

Now we follow through on the geometry and apply Snell's Law once again to find the maximum angle on incident light allowed to satisfy the condition for TIR:

$$\begin{aligned} sin\theta_{max} &= sin(90 - \theta_c) n_{core} \\ &= n_{core} cos(\theta_c) \end{aligned}$$

$$= n_{core}(1 - sin^2\theta_c)^{\frac{1}{2}}$$

Substituting our critical condition then means that the final result simplifies to:

$$sin\theta_{max} = [n_{core}^2 - n_{clad}^2]^{\frac{1}{2}}$$

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